Beauty vs. Earnings: Gender Differences in Earnings and Priorities Over Spousal Characteristics in a Matching Model*

David Bjerk
Claremont McKenna College
Department of Economics
500 E. Ninth St.
Claremont, CA 91711
david.bjerk@cmc.com

January 13, 2008

Abstract

I develop a model of marriage matching where males and females care about two distinct characteristics of their spouse—beauty and earnings. Each individual's beauty is exogenous, but earnings depend on human capital investments made by each individual prior to entering the marriage market. I show that even if males and females constitute ex-ante identical populations, with identical underlying preferences and equal access to human capital investment and labor market opportunities, there can still exist an equilibrium where, on average, one gender invests more in human capital than the other, and moreover, members of one gender are more likely to prioritize beauty over earnings in a spouse, while members of the other gender are more likely to prioritize a potential spouse's earnings over beauty.

JEL codes J12, J16, J24, and D1.

*Thanks to Eric Furstenburg, Dan Hamermesh, Benjamin Karney, and seminar participants at the University of British Columbia for helpful comments on earlier versions of this project. Much of this work was done while I was on a reseach leave at the RAND Corporation.

1 Introduction

It is often perceived that men and women prioritize different characteristics when selecting a mate, with males being more likely to prioritize beauty in a potential spouse, and females being more likely prioritize earnings power in a potential spouse. Anecdotal evidence for this perception abounds, notably in the prevalence of older male celebrities marrying aspiring models and actresses.[1] In more formal studies of this issue, sociologists and psychologists have found that females value good financial prospects in a mate about twice as highly as do men in surveys ranging from the 1930s to the present. Relatedly, these studies also show that across this time period, men rate physical attractiveness as being substantially more important and desirable in a potential mate than do women (Hill, 1945; McGinnis, 1958; Hudson and Henze, 1969; Buss, 1989). Indeed, to quote Buss (1987), "(m)en's greater preference for physically attractive mates is among the most consistently documented psychological sex differences."

More recently, economists Hitsch, Hortacsu, and Ariely (2004) and Fisman, Iyengar, Kamenica, and Simonson (2006) have examined gender differences in priorities over characteristics in dating partners using data from an online dating site and data from a speed dating experiment respectively. In both studies, the results continue to suggest that men place substantially more value on physical appearance in a dating partner than do women, while women place more weight on earnings, or characteristics associated with earnings (e.g. intelligence, race, the wealth of the neighborhood in which one grew up), than do men.

One possible explanation for the above results is simply biological—males may have an innate tendency to favor beauty when choosing their mates, while females may have an innate tendency to favor finding economic security when choosing their mates. Indeed, evolutionary psychologists such as David Buss (1994) have discussed how adaptive evolutionary pressures can account for how these innate tendencies may have arisen. While there may be truth to these biological and evolutionary explanations, as economists, it seems appropriate to further ask whether there also exist complementary or even alternative explanations related to *economic* incentives for why such differing matching priorities across genders arise and persist. Moreover, whether such differences in matching priorities across genders may be related to other outcome differences across

[1] Examples include real-estate tycoon Donald Trump and model Meliana Knauss, musician Billy Joel and model Christie Brinkley, singer Ric Ocasek and model Paulina Porizkova, and director Woody Allen and actress Mia Farrow.

genders, most notably earnings in the labor market.

On a most basic level, I show that under relatively standard economic assumptions, even if genders have identical underlying preferences, genders still can have different priorities over spousal characteristics if there are different earnings outcomes across genders. Intuitively, if individual utility is increasing in both total household earnings as well as spousal beauty, but the *marginal* utility of greater household earnings is diminishing in total household earnings, then the higher an individual's own earnings, the more he or she will value beauty over earnings in a spouse. Alternatively, the lower an individual's own earnings, the more he or she will value earnings over beauty in a spouse. Hence, even if males and females have ex-ante identical underlying preferences, males and females can still have different priorities on average in the marriage market if males generally enter the marriage market with higher earnings trajectories than females.

The primary contribution of the paper, however, is to not only show how gender differences in priorities in the marriage market may arise due differences in earnings outcomes across genders, but also to examine whether differences in earnings outcomes across genders may themselves arise through differences in expectations across genders regarding how different attributes will be valued in the marriage market. In particular, I look at whether expectations of gender differences in earnings outcomes and priorities in the marriage market can be self-fulfilling, even when genders constitute ex-ante identical populations. In other words, if people expect males to earn more in the labor market and be more likely to prioritize beauty over earnings in a spouse than females, then will males indeed turn out to be more likely than females to obtain higher paying jobs and to prioritize beauty over earnings in a spouse?

To examine this question, I construct a model where at the time of the marriage market, individuals can differ along two exogenous dimensions—beauty and earnings. Beauty is important in that an individual is assumed to incur greater utility the greater is the beauty of his or her spouse. Earnings are important in that the utility of a given individual is assumed to be increasing in the total amount of household earnings. The key feature of the model however, is that an individual's earnings outcome depends on the amount he or she invests in human capital, where this investment takes place prior to the marriage market. Therefore, in this environment, there are expected payoffs to human capital investment both in the labor market, in terms of one's own expected earnings, as well as the marriage market, in terms of the expected beauty and earnings level of the spouse one is able to match with.

Two key results arise from this environment. First, marriages will not necessarily be assortative. Specifically, not only might there not be consensus over how individuals on each side of the marriage market should be ranked, but like individuals may often not match with each other. Second, this environment allows for the possibility of multiple equilibria. In particular, given males and females constitute ex-ante identical populations and there is no labor market discrimination, there can exist a symmetric equilibrium where males and females invest similarly in human capital, have similarly distributed earnings outcomes, and have similarly distributed priorities in the marriage market. However, I also show that under certain conditions there also exist asymmetric equilibria, where ex-ante identical populations of males and females invest differently in human capital, have different earnings outcomes on average, and end up with different priorities over spousal characteristics on average. More specifically, if individuals expect members of gender b to have lower earnings on average than members of gender a, individuals will also expect members of gender b to be more likely to prioritize earnings over beauty than members of gender a. I then show that such expectations of differences across genders can cause gender a individuals to indeed find it optimal to invest more in human capital, and subsequently earn more on average than members of gender b, thus causing members of gender b to be more likely to prioritize earnings over beauty in the marriage market than members of gender a, and members of gender a to be more likely to prioritize beauty over earnings in the marriage market than members of gender b.

2 Previous Literature

The model presented below primarily relates to two distinct streams of literature. The first is the theoretical economics literature on gender inequality in the labor market. Like several of the relatively recent theoretical advances in the examination of gender inequality in labor market (Lazear and Rosen, 1990; Francois, 1998; Lommerud and Vagstad, 2000; Albanesi and Olivetti, 2005; Bjerk and Han, 2007), the model developed below generates earnings inequality across genders even when genders are equally productive in the labor market. However, in contrast to these other models, the gender earnings inequality that arises in this model arises despite the fact that employers do not discriminate against either gender.

The second stream of literature related to this paper is the theoretical economics literature on matching in the marriage market. Much of this work stems from Becker's (1973, 1974) work on matching and has expanded in several important dimensions from this initial framework to include the existence of household public goods (Lam, 1988), different timing across genders with respect to when each an individual's quality is revealed (Bergstrom and Bagnoli, 1993)[2], the implications of search frictions (Burdett and Coles, 1997; Burdett and Coles, 1999; Shimer and Smith, 2000; Smith, 2002), and the role of pre-match investment (Burdett and Coles, 2001; Cole, Mailath, and Postlewaite, 2001; Peters and Siow, 2002; Peters, 2005; Nosaka, 2007). A related stream of literature has also considered the role of marriage as a way to mitigate consumption risk (Kotlikoff and Spivak, 1981; Rosenzweig and Stark, 1989; Ogaki and Zhang, 2001).[3]

An important similarity in all of these papers is that the characteristics of each individual that are valued in the marriage market can be summarized by some scalar characteristic. While this characteristic is generally interpreted as the earnings power or wealth each individual brings to the marriage, in some models (particularly in Burdett and Coles, 1997, 1999, 2000; Shimer and Smith, 2000; Shimer, 2002) it can be interpreted as the rank of a given individual in the eyes of the opposite gender, which implicitly assumes that all individuals can be ranked in the marriage market according to some universally agreed upon system that optimally weighs all of the relevant characteristics for each individual. However, regardless of the interpretation, such specifications exclude the possibility that different individuals rank members of the opposite gender differently in equilibrium.

Hess (2004) and Fernandez, Guner, and Knowles (2005) expand the literature on marriage as consumption insurance, and the literature on pre-marriage investment, respectively, by including a notion of love, or match specific utility. In other words, they allow an individual's decision regarding who to marry to depend both the potential spouse's general characteristics, that are valued similarly by all individuals in the marriage market, as well as some other attribute unique to that match. Hence, while a particular individual may have very good general characteristics that are valued in the marriage market, some individuals

[2]In particular, Bergstrom and Bagnoli assume that "The quality of each female is known to all persons when she reaches age 1. The quality of a male does not become public until he reaches age 2." (Bergstrom and Bagnoli, 1993: p 188)

[3]See Mortensen (1988), Roth and Sotomayor (1990), and Rogerson, Shimer, and Wright (2005), as well as the cites therein, for a more comprehensive and general discussion of the two-sided matching and search literature.

may not want to match with him or her due to lack of "love" or match specific utility. While these models lead to some novel and important implications, since love is match specific, these models cannot produce equilibria where individuals differ *systematically* by their characteristics in terms of how they rank individuals of the opposite gender in the marriage market.

Mailath and Postlewaite (2004) develop a model where individuals have two characteristics relevant to the marriage market, their wealth and some other attribute that has no direct impact on anyone's utility or wealth, but has some degree of heritability. They show that even if this other attribute is not productive in any real sense, it may be a valuable asset in the marriage market. The reason being that if people think that individuals with this attribute will be considered desirable mates, and if parents care about their children's well-being, then parents will want their children to have this attribute. Given this attribute is to some degree hereditary, then individuals will want to match with someone with this trait, all else equal. Therefore, individuals with this trait indeed become more desirable mates, fulfilling people's expectations.

An important innovation of Mailath and Postlewaite's model is that it shows that the order in which individuals of one gender rank individuals of the opposite gender may differ for individuals with different characteristics. In particular, they show that under certain conditions, it is possible that a high wealth individual without the attribute will prefer to match with a low wealth individual with the attribute over another high wealth individual without the attribute. While at the same time, a low wealth individual with the attribute will prefer to match with a high wealth individual without the attribute to another low wealth individual with the attribute. Hence, even though this attribute is not inherently of value, it can achieve value in some equilibria, and moreover, can cause individuals to systematically differ in their equilibrium preferences over mates, even though their underlying ex-ante preferences are identical.

The model presented below has some important similarities to many of these papers discussed above. Like Cole, Mailath, and Postlewaite (2001), Peters and Siow (2002), Peters (2005), and Nosaka (2007), the model below focuses on how matching in the marriage market is related to a pre-matching investment that affects earnings outcomes. Moreover, it also has some important similarities to Mailath and Postlewaite (2004). Most notably, in equilibrium, how individuals rank individuals of the opposite gender in the marriage market may differ systematically for individuals with different earnings outcomes. Like in Mailath and Postlewaite (2004), this means marriage may not be perfectly assortative in equilibrium.

A key distinction regarding the results coming from the model developed below and those arising in Mailath and Postlewaite (2004) and the other matching literature cited above, is that unlike those models, equilibrium behavior and outcomes in this model are *not* always symmetric across genders. Specifically, the model developed below shows how males and females can differ both in their behavior and their priorities in the marriage market in equilibrium, even when they are assumed to have ex-ante identical preferences and equal opportunities for investing in human capital and securing high-paying jobs.[4]

3 Earnings Outcomes and Priorities Over Spousal Characteristics

This section highlights one of the crucial aspects of the more complete model developed in the next section by providing an intuitive characterization for why an individual's priorities over spousal characteristics may be related to his or her labor market outcome. In particular, suppose each individual in society is characterized by three observable characteristics—gender, beauty, and earnings. Let an individual's beauty be captured by a parameter $b \in B$ where higher values of b indicate greater beauty. Now, assume that when individuals form a household with an individual of the opposite gender, their personal utility will be increasing in their *total household earnings* (i.e. the sum of each household member's earnings) and the beauty of their spouse. In other words, assume total household earnings are a public good consumed by both household members, while an individual's beauty is a private good consumed only by that individual's spouse. To capture these assumptions more explicitly, let the utility of an individual with an earnings stream of w who marries an individual of the opposite gender with an earnings stream of w^s and beauty level b^s be given by the function $U(w, w^s, b^s) = v(w + w^s) + b^s$, where $v' > 0$.

Now, consider the further assumption that $v'' < 0$, or that individuals incur

[4] It should be said however, that the framework proposed here does not model the role of search frictions. The previously cited work by Burdett and Coles (1997, 1999), Shimer and Smith (2000), and Smith (2002) suggests that allowing for such frictions can have important consequences. To allow for search frictions to play an important role, however, the relatively simple one period matching framework proposed below would have to be adapted to a multi-period setting. Such a modification is beyond the current scope of this paper, but may also provide an important direction for further work.

diminishing marginal utility with respect to total household earnings.[5] Given this assumption of diminishing marginal utility in household earnings, it is easy to show that

$$\frac{\frac{\partial U(w',w^s,b^s)}{db^s}}{\frac{\partial U(w',w^s,b^s)}{dw^s}} > \frac{\frac{\partial U(w'',w^s,b^s)}{db^s}}{\frac{\partial U(w'',w^s,b^s)}{dw^s}} \text{ for all } w' > w''.$$

In words, the marginal rate of substitution between spousal beauty and spousal earnings is increasing in an individual's own earnings stream. Hence, the higher an individual's own earnings, the more he or she is willing to trade off spousal earnings for greater spousal beauty. Given this implication, it is clear that the relative value of earnings versus beauty in the marriage market for one gender depends on the distribution of earnings of the other gender. So, for example, if males generally have higher earnings streams than females, males will generally place a higher value on the beauty of potential spouses relative to the earnings of potential spouses than will females, while females will generally place a higher value on the earnings of potential spouses relative to the beauty of potential spouses than will males.

The above paragraph reveals how individual priorities over spousal characteristics may be fundamentally tied to the individual's labor market outcomes, and relatedly, how it is possible for males and females to have different priorities on average over spousal characteristics even when their underlying preferences with respect to spousal beauty and earnings are ex-ante identical. The model constructed below extends this line of reasoning by examining how differential expectations across genders regarding how earnings versus beauty will be prioritized in the marriage market can lead to differential earnings outcomes across genders, and thereby fulfill initial expectations.

4 Model of Marriage with Pre-Marriage Human Capital Investment and Multiple Matching Characteristics

In order to make things tractable, consider an environment similar to that presented above, but with only two beauty levels, b_h and b_ℓ, where $b_h > b_\ell$, and

[5] This is implicitly very similar to Nosaka's (2007) submodularity assumption over preferences.

two possible earnings levels, $\overline{w}$ and $\underline{w}$, where $\overline{w} > \underline{w}$. Clearly, it will always be the case that all individuals will most prefer to match with a high-beauty (b_h) high-earning ($\overline{w}$) spouse, and least prefer to match with a low-beauty (b_ℓ) low-earning ($\underline{w}$) spouse. However, for the binary distinctions in this model to have any bite, assume that the concavity of the v function is sufficient to ensure that

$$v(\overline{w}+\overline{w}) - v(\overline{w}+\underline{w}) < b_h - b_\ell < v(\overline{w}+\underline{w}) - v(\underline{w}+\underline{w}). \tag{1}$$

Consistent with the intuition discussed in the previous section, the above assumption means that individuals who end up in a high-earning career (i.e. a job with earnings of $\overline{w}$) will prioritize beauty over earnings in a spouse, while those who end up in a low-earning career will prioritize earnings over beauty in a spouse. More specifically, if we let $x \succ y$ indicate that an individual strictly prefers option x to option y, and let any individual i be characterized by a tuple $\{w_i, b_i\}$, where w_i is individual i's earnings level and b_i is individual i's beauty level, the assumption in equation (1) directly implies that each individual who ends up with a high-earnings job will have marriage market priorities such that

$$\{\overline{w}, b_h\} \succ \{\underline{w}, b_h\} \succ \{\overline{w}, b_\ell\} \succ \{\underline{w}, b_\ell\}$$

and each individual that ends up with a low-earning job will have marriage market priorities such that

$$\{\overline{w}, b_h\} \succ \{\overline{w}, b_\ell\} \succ \{\underline{w}, b_h\} \succ \{\underline{w}, b_\ell\}.$$

Next, assume that while an individual's beauty is exogenous, an individual's earnings outcome depends on how much the individual invests in human capital.[6] Namely, assume that by investing in human capital, an individual increases his or her probability of ending up with a high-earnings career. In particular, suppose an individual can ensure a probability of π of landing a high-earning career by paying a utility cost equal to $c(\pi)$.

The timing of the model unfolds as follows. Individuals first simultaneously decide how much to invest in labor market human capital. After making this investment, individuals realize which earnings track they land on, after which

[6] This is in contrast to Burdett and Coles (2001) model a marriage matching environment where individuals can invest in beauty. However, in their model individuals can not invest in labor market human capital that pays off in higher expected earnings, and moreover, earnings are not valued in the marriage market.

they match with someone of the opposite gender in the marriage market. There are a variety of ways to model how matches form in the marriage market. In order to avoid excessive complication, in this analysis I'll use a version of the "deferred acceptance algorithm" first formalized by Gale and Shapley (1962), and later analyzed by Roth (1984) and Roth and Vande Vate (1990), to study the *marriage* problem.

Specifically, in the version used here, we will label the two genders a and b. At the beginning of the first "round," each gender a individual proposes to a gender b individual at the top of his/her preference list (i.e. a gender b individual such that there are no other gender b individuals he/she strictly prefers to the individual he/she proposed to). Each gender b individual who receives a proposal in this first round then picks his/her most preferred gender a proposer and rejects all others. If there are several gender a proposers between whom he/she is indifferent, but these gender a individuals are preferred to all the other proposers, he/she will randomly select one from this preferred set and reject all others. If a gender b individual receives only one proposal he/she "picks" this proposer. All gender a individuals who are rejected in the first round then move on to a second round and propose to their next most preferred gender b individual. Gender b individuals who received no proposals in the first round but receive one or more proposals in this second round act exactly like gender b individuals who received one or more proposals in the first round. Gender b individuals who received one or more proposals in the first round will reject all second round proposals they receive if they weakly prefer the proposer they selected in the first round to all their second round proposers. However, if a gender b individual receives a proposal from a gender a individual in the second round who he/she strictly prefers to the proposer who he/she selected in the first round, he/she chooses this proposer and rejects the proposer he/she chose in the previous round as well as his/her other proposers in this round. This process keeps repeating until there are no gender a individuals left to make proposals (i.e. no gender a individual who are rejected), at which time individuals "marry" the partner they are currently matched with.

The key feature of this process is that it leads to *stable* matchings, in the sense that this process cannot lead to two married couples such that the gender a individual in one marriage strictly prefers the gender b individual in the other marriage, and this gender b individual in the other marriage also strictly prefers this gender a individual to his/her current spouse.

Given the preferences discussed above, and if we denote the realized fraction of gender g beauty level j individuals who obtain a high-earnings job as π_j^g, then we can directly derive the probabilties each type of match will occur given any $\pi_h^a \geq \pi_h^b$ and $\pi_\ell^a \geq \pi_\ell^b$. Specifically, letting $p_j^g(\{w,b\}|w_i)$ denote the probability that an individual i of gender g and beauty level j with earnings w_i matches with a spouse of type-$\{w,b\}$, then we get the following Lemma:

Lemma 1 *Given any realized* $\{\pi_h^a, \pi_h^b, \pi_\ell^a, \pi_\ell^b\}$ *such that* $\pi_h^a \geq \pi_h^b$ *and* $\pi_\ell^a \geq \pi_\ell^b$, *and the matching process described above, the match type probabilities for each beauty level in each gender are given by:*

(1) High-Beauty Gender a Individuals

$$\begin{array}{ll}
p_h^a(\{\overline{w},b_h\}|\overline{w}) = \frac{\pi_h^b}{\pi_h^a} & p_h^a(\{\overline{w},b_h\}|\underline{w}) = 0 \\
p_h^a(\{\underline{w},b_h\}|\overline{w}) = \frac{\pi_h^a-\pi_h^b}{\pi_h^a} & p_h^a(\{\underline{w},b_h\}|\underline{w}) = 0 \\
p_h^a(\{\overline{w},b_\ell\}|\overline{w}) = 0 & p_h^a(\{\overline{w},b_\ell\}|\underline{w}) = \begin{cases} 1 \text{ if } \pi_\ell^b > 0 \\ 0 \text{ otherwise} \end{cases} \\
p_h^a(\{\underline{w},b_\ell\}|\overline{w}) = 0 & p_h^a(\{\underline{w},b_\ell\}|\underline{w}) = \begin{cases} 0 \text{ if } \pi_\ell^b > 0 \\ 1 \text{ otherwise} \end{cases}
\end{array}$$

(2) Low-Beauty Gender a Individuals

$$\begin{array}{ll}
p_\ell^a(\{\overline{w},b_h\}|\overline{w}) = 0 & p_\ell^a(\{\overline{w},b_h\}|\underline{w}) = 0 \\
p_\ell^a(\{\underline{w},b_h\}|\overline{w}) = \frac{1-\pi_h^a}{\pi_\ell^a} & p_\ell^a(\{\underline{w},b_h\}|\underline{w}) = 0 \\
p_\ell^a(\{\overline{w},b_\ell\}|\overline{w}) = \frac{\pi_\ell^b-(1-\pi_h^a)}{\pi_\ell^a} & p_\ell^a(\{\overline{w},b_\ell\}|\underline{w}) = 0 \\
p_\ell^a(\{\underline{w},b_\ell\}|\overline{w}) = \frac{\pi_\ell^a-\pi_\ell^b}{\pi_\ell^a} & p_\ell^a(\{\underline{w},b_\ell\}|\underline{w}) = 1
\end{array}$$

(3) High-Beauty Gender b Individuals

$$\begin{array}{ll}
p_h^b(\{\overline{w},b_h\}|\overline{w}) = 1 & p_h^b(\{\overline{w},b_h\}|\underline{w}) = \frac{\pi_h^a-\pi_h^b}{1-\pi_h^b} \\
p_h^b(\{\underline{w},b_h\}|\overline{w}) = 0 & p_h^b(\{\underline{w},b_h\}|\underline{w}) = 0 \\
p_h^b(\{\overline{w},b_\ell\}|\overline{w}) = 0 & p_h^b(\{\overline{w},b_\ell\}|\underline{w}) = \frac{1-\pi_h^a}{1-\pi_h^b} \\
p_h^b(\{\underline{w},b_\ell\}|\overline{w}) = 0 & p_h^b(\{\underline{w},b_\ell\}|\underline{w}) = 0
\end{array}$$

(4) Low-Beauty Gender b Individuals

$$\begin{array}{ll}
p_\ell^b(\{\overline{w},b_h\}|\overline{w}) = 0 & p_\ell^b(\{\overline{w},b_h\}|\underline{w}) = 0 \\
p_\ell^b(\{\underline{w},b_h\}|\overline{w}) = \frac{1-\pi_h^a}{\pi_\ell^b} & p_\ell^b(\{\underline{w},b_h\}|\underline{w}) = 0 \\
p_\ell^b(\{\overline{w},b_\ell\}|\overline{w}) = \frac{\pi_\ell^b-(1-\pi_h^a)}{\pi_\ell^b} & p_\ell^b(\{\overline{w},b_\ell\}|\underline{w}) = \frac{\pi_\ell^a-\pi_\ell^b}{1-\pi_\ell^b} \\
p_\ell^b(\{\underline{w},b_\ell\}|\overline{w}) = 0 & p_\ell^b(\{\underline{w},b_\ell\}|\underline{w}) = \frac{1-\pi_\ell^a}{1-\pi_\ell^b}
\end{array}$$

Proof. In Appendix. ■

Clearly, an analogous Lemma could be stated for any any realized $\{\pi_h^a, \pi_h^b, \pi_\ell^a, \pi_\ell^b\}$

such that $\pi_h^a \leq \pi_h^b$ and $\pi_\ell^a \leq \pi_\ell^b$ simply by switching all the gender labels.[7]

The intuition for the Lemma 1 is as follows. First recall that all individuals with high-earnings will most prefer to match with a high-beauty high-earner, followed by a high-beauty low-earner, followed by a low-beauty high-earner, followed by a low-beauty low-earner. Alternatively, all individuals with low-earnings will most prefer to match with a high-beauty high-earner, followed by a low-beauty high-earner, follwed by a high-beauty low-earner, followed by a low-beauty low-earner.

Given there is no cost to making a proposal, the above preferences mean optimal marriage market behavior for all gender a individuals is to first propose to a high-beauty high-earner. If rejected, each gender a individual should propose to any high-beauty high-earning b who does not have a high-beauty high-earning suitor. When there are no remaining high-beauty high-earning b's without high-beauty high-earning suitors, high-earning a's should then propose to high-beauty low-earning b's. Those high-earning a's who are rejected by all high-beauty b's should then propose to low-beauty high-earning b's until they are no longer rejected. Alternatively, low-earning a's who are rejected by all high-earning high-beauty b's should then propose to low-beauty high-earning b's. Those low-earning a's who are rejected by all high-earning b's should then propose to high-beauty low-earning b's until they are no longer rejected. Finally, any a's who are rejected by all high-beauty and high-earning b's should propose to low-beauty low-earning b's.

On the other side of the marriage market, a gender b individual should accept a proposal from a low-beauty low-earning individual only if (s)he receives no other proposals. A high-earning b should accept a proposal from a low-beauty high-earning a only if all other proposals are from low-beauty low-earners, and accept a proposal from a high-beauty low-earning a only if all other proposals are from low-beauty a's. Alternatively, a low-earning b should accept a proposal from a high-beauty low-earning a only if all other proposals are from low-beauty low-earners, and accept a proposal from a low-beauty high-earning a only if all other proposals are from low earners. Finally, a gender b individual should accept any proposal from a high-beauty high-earning a as long as they have not already accepted a proposal from a different high-beauty high-earning a.

[7] Analogous Lemmas could also be derived for any realized $\{\pi_h^a, \pi_h^b, \pi_\ell^a, \pi_\ell^b\}$ such that $\pi_h^a \geq \pi_h^b$ and $\pi_\ell^a \leq \pi_\ell^b$, or such that $\pi_h^a \leq \pi_h^b$ and $\pi_\ell^a \geq \pi_\ell^b$. However, at this point, I cannot prove that any such realizations can arise in the type of equilibria considered in this paper. Therefore, I do not present the match probabilities for such cases here.

These strategies mean that high-beauty high-earning types match with each other to the greatest extent possible. However, if $\pi_h^a > \pi_h^b$, this will mean that while all high-beauty high-earnings gender b's will form such matches, only a fraction $\frac{\pi_h^b}{\pi_h^a}$ of high-beauty high-earnings gender a's will be able to form such matches, with the remaining fraction $\frac{\pi_h^a-\pi_h^b}{\pi_h^a}$ having to form matches with their next most preferred type, namely high-beauty but low-earning gender b's. This means a fraction $\frac{\pi_h^a-\pi_h^b}{1-\pi_h^b}$ of the high-beauty but low-earning gender b's will match with high-beauty high-earning a's, leaving a fraction $\frac{1-\pi_h^a}{1-\pi_h^b}$ yet unmatched. This remaining fraction of high-beauty but low-earning gender b's willl then match with low-beauty but high-earning gender a's. This still leaves a fraction $\frac{1-\pi_h^a}{\pi_\ell^a}$ of low-beauty but high-earning gender a's unmatched to a high-beauty b. Alternatively, all high-beauty low-earning a's will match with low-beauty high-earning b's, which will mean a fraction $\frac{1-\pi_h^a}{\pi_\ell^b}$ of low-beauty high-earning b's will form such matches, but will still leave a fraction $\frac{\pi_\ell^b-(1-\pi_h^a)}{\pi_\ell^b}$ of low-beauty high-earning b's unmatched to a high-beauty a.

These low-beauty high-earning individuals of both genders who do not match with a high-beauty individual of the opposite gender will then match with each other. However, if $\pi_\ell^a > \pi_\ell^b$, there will be more such unmatched low-beauty high-earning a's than b's. Therefore, while all low-beauty high-earning b's who don't match with a high-beauty spouse will match with a low-beauty but high-earning spouse, only a fraction $\frac{\pi_\ell^b-(1-\pi_h^a)}{\pi_\ell^a}$ of low-beauty high-earning a's will do so. The remaining fraction $\frac{\pi_\ell^a-\pi_\ell^b}{\pi_\ell^a}$ of low-beauty high-earning a's will match with low-beauty low-earning b's, meaning a fraction $\frac{\pi_\ell^a-\pi_\ell^b}{1-\pi_\ell^b}$ of low-beauty low-earning b's will match with low-beauty high-earning a's. Finally, all low-beauty low-earning a's will propose to the remaining fraction $\frac{1-\pi_\ell^a}{1-\pi_\ell^b}$ of low-beauty low-earning b's.

4.1 Equilibria with Binary Human Capital Investment and Certain Payoffs

Let us first consider the simplest case where the human capital investment is binary, meaning an individual either does or does not invest in human capital, and the labor market payoff to such an investment is certain. Namely, if an individual invests in human capital the probability of obtaining a high-earnings job equals one, while probability of obtaining a high-earning job without the human capital investment is zero. In terms of the notation described above, the only human capital choice possible is setting $\pi = 1$ at a cost of $c(1)$, which we can denote simply as c.

Given this simple structure, an expected utility maximizing individual of gender g and beauty type-j will invest in human capital (i.e. set $\pi_j^g = 1$) if and only if $E[U|invest] > E[U|no - invest]$, or

$$\begin{aligned}
&\widehat{p}_j^g(\{\overline{w}, b_h\}|\overline{w})[v(\overline{w}+\overline{w})+b_h] + \widehat{p}_j^g(\{\underline{w}, b_h\}|\overline{w})[v(\overline{w}+\underline{w})+b_h] \\
&+ \widehat{p}_j^g(\{\overline{w}, b_\ell\}|\overline{w})[v(\overline{w}+\overline{w})+b_\ell] + \widehat{p}_j^g(\{\underline{w}, b_\ell\}|\overline{w})[v(\overline{w}+\underline{w})+b_\ell]\big) - c > \\
&\widehat{p}_j^g(\{\overline{w}, b_h\}|\underline{w})[v(\underline{w}+\overline{w})+b_h] + \widehat{p}_j^g(\{\underline{w}, b_h\}|\underline{w})[v(\underline{w}+\underline{w})+b_h] \\
&+ \widehat{p}_j^g(\{\overline{w}, b_\ell\}|\underline{w})[v(\underline{w}+\overline{w})+b_\ell] + \widehat{p}_j^g(\{\underline{w}, b_\ell\}|\underline{w})[v(\underline{w}+\underline{w})+b_\ell]\big),
\end{aligned} \tag{2}$$

where $\widehat{p}_j^g(\{w, b\}|w)$ denotes the expectations of an individual of gender g and beauty level j for matching with a spouse of type-$\{w, b\}$ conditional on his or her realized earnings w.

In any equilibrium characterization of this environment I assume individuals act optimally subject to their beliefs. Moreover, in order to provide a plausible model of long-run behavior, these beliefs must be consistent with the resulting truth. Therefore, the relevant equilibrium concept for this environment is a *Perfect Bayesian Equilibrium (PBE)*, where all individuals act optimally given their beliefs, and each individual's beliefs are consistent with Bayes' rule given each other player acts optimally. A *PBE* in this environment will be characterized by a four-tuple $\{\pi_h^m, \pi_h^f, \pi_\ell^m, \pi_\ell^f\}$ that for each beauty level j/gender g combination, $\pi_j^g = 1$ if an only if equation (2) holds given $\widehat{p}_j^g(\{w', b_j\}|w'') = p_j^g(\{w', b_j\}|w'')$, where $p_j^g(\{w', b_j\}|w'')$ is the true probability that an individual of gender g and beauty level j matches with an individual of type-$\{w', b_j\}$ given he or she ends up with earnings of w'' and the fraction of each gender beauty type that ends up with high-earnings is given by the four-tuple $\{\pi_h^a, \pi_h^b, \pi_\ell^a, \pi_\ell^b\}$.

If we define $\Delta v(\overline{w})$ to equal $v(\overline{w}+\overline{w}) - v(\overline{w}+\underline{w})$, Δb to equal $b_h - b_\ell$, and $\Delta v(\underline{w})$ to equal $v(\overline{w}+\underline{w}) - v(\underline{w}+\underline{w})$, we get the following Proposition.

Proposition 1 *If $\Delta v(\overline{w}) < c < \Delta v(\underline{w})$, then there exists an asymmetric equilibrium where $\pi_h^a = 1, \pi_h^b = 0, \pi_\ell^a = 1$, and $\pi_\ell^b = 0$.*

Proof. In Appendix. ■

Clearly, given the two gender groups are ex-ante identical, when $\Delta v(\overline{w}) < c < \Delta v(\underline{w})$, then there also exists another asymmetric equilibrium where $\pi_h^a =$

$0, \pi_h^b = 1, \pi_\ell^a = 0$, and $\pi_\ell^b = 1$. In words, Proposition 1 states that when the cost of the human capital investment is such that it is only ex-post optimal to invest in human capital when one's spouse turns out to be a low-earner, then in equilibrium, only members of one gender should make this investment. This result is by no means surprising, as it simply reveals that specialization should occur in the economy when it is not efficient for everyone to invest in human capital, and one way to coordinate this specialization is for members of one gender to make the investment but not members of the other.

Furthermore, it will also be true that in an equilibrium of the type described in Proposition 1, marriage will be assortative in beauty type (this can be seen by plugging in $\pi_h^a = 1, \pi_h^b = 0, \pi_\ell^a = 1$, and $\pi_\ell^b = 0$ into the probability expressions in Lemma 1). However, the interesting feature that comes out of the type of equilibrium described in Proposition 2 is that the gender groups will have different equilibrium *rankings* over spousal types. In particular, since type-a individuals will all have high-earnings (since they all invest), in equilibrium, if asked how they rank potential spouses they will say they prioritize beauty over earnings (from assumption (1)). Alternatively, since type-b individuals will all have low-earnings (since none of them invest), in equilibrium, if asked how they rank potential spouses, they will say they priortize earnings over beauty. This is true even though the two gender groups have identical ex-ante preferences and labor market opportunites.

One question that arises is why is this other individual characteristic b referred to as beauty since it is simply some characteristic of a potential spouse that individuals care about other than the potential spouse's earnings. In this way, one could also interpret it as domestic skills, for example, such as cooking ability or cleaning ability. Why I interpret it as beauty is twofold. First, it must be a characteristic that is directly observable prior to marriage. This is arguably more true for something like physical beauty than domestic skills. Second, and more importantly, the goal of this paper is to understand the extent to which economic forces can help explain why males are more likely to prioritize beauty over earnings in a spouse, while females are more likely to prioritize earnings over beauty. As discussed above, such gender differences in priorties over spousal characteristics can come out of this model when b is interpreted as physical beauty.

4.2 Equilibria with Continuous Human Capital Investment and Uncertainty

Two key concerns regarding the environment discussed above is what would happen if there was some uncertainty regarding how the human capital investment translates into earnings, and what would happen if people could invest in human capital in a continuous way (rather than in the lumpy binary way assumed above)? Given uncertainty, it will be true that some group-a individuals will end up having low-earnings. If individuals are then allowed to invest "a little bit" in human capital, group-b individuals may choose to do so as they are no longer guaranteed to end up in a household with a spouse who is a high-earner. This in turn lowers group-a individuals' incentive to invest as much in human capital, because now there might up matching with a group-b spouse with high-earnings. This section explores this generalization of the model.

Again, denote the utility cost of investing in human capital as $c(\pi)$, where π is the probability of landing a high-earnings career. However, now assume $c'(\pi) > 0$ and $c''(\pi) > 0$ for all $\pi \in (0,1)$. Intuitively, assume the cost of increasing one's probability of landing a high-earnings career is increasing at an increasing rate. Furthermore, assume that $c'(0) = 0$ and $\lim_{\pi \to 1} c'(\pi) = \infty$.

In the context of this environment, an expected utility maximizing individual will choose π so as to maximize the following equation:

$$\begin{aligned} &\pi \left(\widehat{p}_j^g(\{\overline{w}, b_h\}|\overline{w})[v(\overline{w}+\overline{w})+b_h] + \widehat{p}_j^g(\{\underline{w}, b_h\}|\overline{w})[v(\overline{w}+\underline{w})+b_h]\right. \\ &\quad \left. + \widehat{p}_j^g(\{\overline{w}, b_\ell\}|\overline{w})[v(\overline{w}+\overline{w})+b_\ell] + \widehat{p}_j^g(\{\underline{w}, b_\ell\}|\overline{w})[v(\overline{w}+\underline{w})+b_\ell]\right) \\ &+(1-\pi)\left(\widehat{p}_j^g(\{\overline{w}, b_h\}|\underline{w})[v(\underline{w}+\overline{w})+b_h] + \widehat{p}_j^g(\{\underline{w}, b_h\}|\underline{w})[v(\underline{w}+\underline{w})+b_h]\right. \\ &\quad \left. + \widehat{p}_j^g(\{\overline{w}, b_\ell\}|\underline{w})[v(\underline{w}+\overline{w})+b_\ell] + \widehat{p}_j^g(\{\underline{w}, b_\ell\}|\underline{w})[v(\underline{w}+\underline{w})+b_\ell]\right) - c(\pi). \end{aligned} \tag{3}$$

where $\widehat{p}_j^g(\{w, b\}|w)$ again denotes the expectations of an individual of gender g and beauty level j for matching with a spouse of type-$\{w, b\}$ conditional on his or her realized earnings w. The first two lines of the above expression show the expected utility the individual incurs if he or she lands a high-earnings career times the probability he or she lands a high-earnings career. The bottom two lines show the expected utility the individual incurs if he or she ends up in a low-earnings career times the probability he or she ends up in a low-earning career, minus the utility cost of the human capital investment he or she makes.

Given an individual's expectations and the fact that $c(\pi)$ is an increasing convex function of π, we can solve for optimal human capital investment given expectations simply by deriving the first order condition from equation (3). Doing so we get the following expression that characterizes optimal human capital investment π_j^g for an individual of beauty type j and gender g given his or her beliefs:

$$\begin{aligned} c'(\pi_j^g) = &\big(\widehat{p}_j^g(\{\overline{w}, b_h\}|\overline{w})[v(\overline{w}+\overline{w})+b_h] + \widehat{p}_j^g(\{\underline{w}, b_h\}|\overline{w})[v(\overline{w}+\underline{w})+b_h] \\ &+ \widehat{p}_j^g(\{\overline{w}, b_\ell\}|\overline{w})[v(\overline{w}+\overline{w})+b_\ell] + \widehat{p}_j^g(\{\underline{w}, b_\ell\}|\overline{w})[v(\overline{w}+\underline{w})+b_\ell]\big) \\ &- \big(\widehat{p}_j^g(\{\overline{w}, b_h\}|\underline{w})[v(\underline{w}+\overline{w})+b_h] + \widehat{p}_j^g(\{\underline{w}, b_h\}|\underline{w})[v(\underline{w}+\underline{w})+b_h] \\ &+ \widehat{p}_j^g(\{\overline{w}, b_\ell\}|\underline{w})[v(\underline{w}+\overline{w})+b_\ell] + \widehat{p}_j^g(\{\underline{w}, b_\ell\}|\underline{w})[v(\underline{w}+\underline{w})+b_\ell]\big). \end{aligned} \tag{4}$$

Given that in a PBE it must be the case that $\widehat{p}_j^g(\{w', b_j\}|w'') = p_j^g(\{w', b_j\}|w'')$, where $p_j^g(\{w', b_j\}|w'')$ is the true probability that an individual of gender g and beauty level j matches with an individual of type-$\{w', b_j\}$ given he or she ends up with earnings of w'', we can substitute in the values from Lemma 1 into equation (4) to see that for any equilibrium where $\pi_h^a \geq \pi_h^b$ and $\pi_\ell^a \geq \pi_\ell^b$, there must exist a four-tuple $\{\pi_h^a, \pi_\ell^a, \pi_h^b, \pi_\ell^b\}$ that simultaneously solves the following four equations

$$c'(\pi_h^a) = \frac{\pi_h^b}{\pi_h^a}[v(\overline{w}+\overline{w})+b_h] + \frac{\pi_h^a - \pi_h^b}{\pi_h^a}[v(\overline{w}+\underline{w})+b_h] - [v(\overline{w}+\underline{w})+b_\ell],$$

$$c'(\pi_h^b) = [v(\overline{w}+\overline{w})+b_h] - \frac{\pi_h^a - \pi_h^b}{1-\pi_h^b}[v(\overline{w}+\underline{w})+b_h] - \frac{1-\pi_h^a}{1-\pi_h^b}[v(\overline{w}+\underline{w})+b_\ell],$$

$$\begin{aligned} c'(\pi_\ell^a) \quad = \quad & \frac{1-\pi_h^a}{\pi_\ell^a}[v(\overline{w}+\underline{w})+b_h] + \frac{\pi_\ell^b - (1-\pi_h^a)}{\pi_\ell^a}[v(\overline{w}+\overline{w})+b_\ell] \\ & + \frac{\pi_\ell^a - \pi_\ell^b}{\pi_\ell^a}[v(\overline{w}+\underline{w})+b_\ell] - [v(\underline{w}+\underline{w})+b_\ell], \end{aligned}$$

$$\begin{aligned} c'(\pi_\ell^b) \quad = \quad & \frac{1-\pi_h^a}{\pi_\ell^f}[v(\overline{w}+\underline{w})+b_h] + \frac{\pi_\ell^b - (1-\pi_h^a)}{\pi_\ell^b}[v(\overline{w}+\overline{w})+b_\ell] \\ & - \frac{\pi_\ell^a - \pi_\ell^b}{1-\pi_\ell^b}[v(\overline{w}+\underline{w})+b_\ell] - \frac{1-\pi_\ell^a}{1-\pi_\ell^b}[v(\underline{w}+\underline{w})+b_\ell]. \end{aligned}$$

Again defining $\Delta v(\overline{w})$ to equal $v(\overline{w}+\overline{w}) - v(\overline{w}+\underline{w})$, Δb to equal $b_h - b_\ell$, and $\Delta v(\underline{w})$ to equal $v(\overline{w}+\underline{w}) - v(\underline{w}+\underline{w})$, then with some algebra we can simplify above four equations to be

$$c'(\pi_h^a) = \frac{\pi_h^b}{\pi_h^a}\Delta v(\overline{w}) + \Delta b, \tag{5}$$

$$c'(\pi_h^b) = \Delta v(\overline{w}) + \frac{1-\pi_h^a}{1-\pi_h^b}\Delta b, \tag{6}$$

$$c'(\pi_\ell^a) = \Delta v(\underline{w}) + \frac{1-\pi_h^a}{\pi_\ell^a}(\Delta b - \Delta v(\overline{w})) + \frac{\pi_\ell^b}{\pi_\ell^a}\Delta v(\overline{w}), \tag{7}$$

$$c'(\pi_\ell^b) = \frac{1-\pi_\ell^a}{1-\pi_\ell^b}\Delta v(\underline{w}) + \frac{1-\pi_h^a}{\pi_\ell^a}(\Delta b - \Delta v(\overline{w})) + \Delta v(\overline{w}). \tag{8}$$

Therefore, a PBE is characterized by a four-tuple $\{\pi_h^a, \pi_\ell^a, \pi_h^b, \pi_\ell^b\}$, where $\pi_h^a \geq \pi_h^b$ and $\pi_\ell^a \geq \pi_\ell^b$, that simultaneously solves equations (5)-(8).

Not surprisingly, given the two genders groups are assumed to be ex-ante identical populations, there exists a symmetric PBE where gender b individuals invest in human capital similarly to their analogous gender a counterparts. This is stated explicitly in Proposition 1 below.

Proposition 2 *There exists a symmetric PBE where* $\pi_h^a = \pi_h^b$ *and* $\pi_\ell^a = \pi_\ell^b$.

Proof. In Appendix. ■

In this symmetric PBE it is obviously the case that gender a individuals and gender b individuals have the same distribution of earnings outcomes on average, and given similar distribution of earnings outcomes, both gender groups also have a similar distribution of priorities over spousal characteristics.

As motivated by the introduction however, the key interest of this analysis is to determine whether there can also exist asymmetric equilibria where individuals in the two gender groups behave differently even though they constitute ex-ante identical populations. In order to examine whether this can be true, let us define π^* to be such that $c'(\pi^*) = \Delta b$. Given this definition, I can now state sufficient conditions for the existence of asymmetric equilibria in the Proposition below.

Proposition 3 *Given π^* as defined above:*

1. If there exists a $\pi < \pi^*$ such that $c'(\pi) \geq \Delta v(\overline{w}) + \frac{1-\pi^*}{1-\pi}\Delta b$, then there exists at least one asymmetric equilibrium where $\pi_h^a > \pi^* > \pi_h^b$ and $\pi_\ell^a \geq \pi_\ell^b$.

2. If there exists a $\pi < \pi^*$ such that $c'(\pi) \geq \Delta v(\overline{w}) + \frac{1-\pi^*}{1-\pi}\Delta v(\underline{w}) + \frac{1-\pi^*}{\pi^*}(\Delta b - \Delta v(\overline{w}))$, then there exists at least one asymmetric equilibrium where $\pi_h^a > \pi^* > \pi_h^b$ **and** $\pi_\ell^a > \pi^* > \pi_\ell^b$.

Proof. In Appendix. ■

The above proposition shows that there can indeed exist asymmetric equilibria in this environment, even when the two gender groups constitute ex-ante identical populations. In such asymmetric equilibria, gender a individuals invest more in human capital than gender b individuals, and therefore, are more likely to land a high-earning job and earn more on average than gender b individuals. Moreover, given more gender a individuals than gender b individuals land high-earning jobs in such asymmetric equilibria, then from the discussion above following equation (1), we know that a higher fraction of gender a individuals than gender b individuals will prioritize *beauty over earnings* in the marriage market, while a higher fraction of gender b individuals than gender a individuals will prioritize *earnings over beauty* in the marriage market. Hence, Proposition 3 shows that, in this environment, it is indeed possible for gender groups to not only have different average earnings levels in equilibrium, but also different average priorities over spousal characteristics in equilibrium, even if the gender groups are ex-ante identical.

Another important thing to note about an asymmetric equilibrium in this enhanced environment with a continous choice over human capital invesment and uncertainty is that marriage will *no longer be perfectly positively or perfectly negatively assortative in beauty or earnings.* Namely, there will be some low-beauty a's who match with high-beauty b's and vice versa, and likewise, several high-earning a's who match with low-earning b's and vice versa. However, there will also be several high-beauty high-earning individuals individuals who match with each other, and several low-beauty low-earning individuals who match with each other. Moreover, as discussed above, the primary implication coming from Assumption (1) is that those with high-earnings will have different priorities over spousal characteristics than those with low-earnings. This leads to a a third implication to come out of Proposition 3—namely that in an asymmetric equilibrium, since there is within gender heterogeneity in earnings outcomes, there should also be *within* gender heterogeneity in priorities over spousal characteristics.

As Proposition 3 also reveals, however, the existence of such asymmetric equilibria is not guaranteed generically. Specifically, the sufficient conditions laid out in Proposition 3 that guarantee the existence of such equilibria are not innocuous. Intuitively, these conditions amount to saying that $c(\pi)$ must increase relatively increasing rate in π, or that the marginal cost of increasing expected earnings must rise quite rapidly, and moreover, that this marginal cost quickly becomes relatively large compared to $\Delta v(\overline{w})$, meaning there is only a relatively small increase in utility in going from household with one high-earner and one low-earner to a household with two high earners. If we let $f(\pi) = \Delta v(\overline{w}) + \frac{1-\pi^*}{1-\pi}\Delta b$, then this condition required for point 1 of Proposition 3 to hold can be graphically depicted by Figure 1. The graphical depiction for the condition required for point 2 of Proposition 1 to hold is analogous.[8] As this figure shows, the conditions for Proposition 3 to hold essentially come down to the cost of human capital having to increase at a sufficiently increasing rate.

While these sufficient conditions in Proposition 3 are quite technical, the underlying intuition behind the existence asymmetric equilibria is actually quite straightforward. Specifically, suppose gender b individuals expect a high fraction of gender a individuals to end up in high-earning careers. Then, gender b individuals will expect to have a high likelihood of ending up with a high-earning spouse regardless of their own human capital investment. Moreover, such expectations mean gender b individuals will also not expect their own earnings outcomes to have much of an impact on their own marriage prospects, as they will expect potential gender a partners to generally prioritize beauty. Therefore, *if there is a high marginal cost of increasing the likelihood of getting a high-earning career, and individuals incur very little marginal benefit to having a high-earning career given they end up with a high-earning spouse*, then gender b individuals may find it optimal to "free-ride" on gender a individuals given they expect a high fraction of gender a individuals to have high-earning careers.

On the other hand, the opposite will be true for gender a individuals. Specif-

[8] The increasing convex shape of the $f(\pi)$ function in Figure 1 can be confirmed by taking the first and second derivatives with respect to π of the right-hand side of the equation in point 1 of Proposition 3. A similar result will hold when taking the first and second derivatives of the condition in point 2 of Proposition 3.

ically, suppose gender a individuals expect a low fraction of gender b individuals will end up in high-earning careers. Then, not only will gender b individuals expect that obtaining a high-earning career will be very beneficial financially, since they will expect to end up in a household with two low-earners otherwise, but they will also expect that such a high-earning career will be greatly valued by potential gender b spouses in the marriage market. Hence, even if the marginal cost of increasing the likelihood of landing a high-earning job increases quickly, gender a individuals may still have a strong incentive to make the investments necessary to ensure a relatively high likelihood of obtaining a high-earning job given they expect few gender b individuals will end up in such high-earning jobs.

The issue of free-riding leads to the final implication of the model, detailed in the following Proposition.

Proposition 4 *When both a symmetric and asymmetric equilibria exist, a higher fraction of* ***both*** *gender groups end up in high-earnings careers in the symmetric equilibrium than in an asymmetric equilibrium.*

Proof. In Appendix. ■

It is not surprising that in an asymmetric equilibrium where gender b individuals invest less in human capital than their gender a counterparts, these gender b individuals invest less in human capital and earn less on average than they would in the symmetric equilibrium. However, initial intuition may suggest that in such an asymmetric equilibrium, gender a individuals would actually invest *more* in human capital and earn more on average than they would in the symmetric equilibrium. Proposition 4 reveals that this is not the case though. The intuition for this result is related to the intuition discussed above, where in an asymmetric equilibrium, gender b individuals *free-ride* off of the human capital investments made by gender a individuals. This lowers the benefit to gender a individuals of investing in human capital because the value of marriage is somewhat lowered by the fact that only a small fraction of gender b individuals end up bringing high-earnings to the household.

This result is very similar to that of Nosaka (2007), who also develops a model of marriage with pre-match investment and finds the existence of both symmetric and asymmetric equilibria between ex-ante identical genders, with the characteristic that members of both genders invest less in an asymmetric equilibrium than a symmetric equilibrium. Interestingly, while Nosaka's model has only one individual trait valued in the marriage market, the basic forces behind this particular result are essentially the same in Nosaka's as in this paper. As discussed by Nosaka, specialization and competition are both important elements to the interaction between human capital investment and marriage. In particular, specialization is important in the sense that it is only ex-post optimal for an individual to have high-earnings if the individual married a low-earnings indivdiual. This specialization effect lowers the incentive to invest in human capital for one side of the marriage market when they expect they anticipate high levels of human capital investment from the other side (and vice versa). Competition is important in the sense that when individuals anticipate high levels of human capital investment from other members of their own gender, they have a greater incentive to invest as well to effectively compete for a desirable spouse. Like in Nosaka's model, this competition motivation becomes stronger when there is high level of human capital investment on both sides of the marriage market. The interaction between this competition effect and the interaction effect are what allow for multiple equilibria. However, this paper builds on this result developed by Nosaka, by showing how these differing investment rates across equilibria can not only help us understand earnings inequality across genders, but also differences across genders in priorities over spousal characteristics.

Finally, a direct implication of the above Proposition 4 is that *aggregate income is higher in the symmetric equilibrium than an asymmetric equilibrium.* Hence, in this environment, not only is there earnings inequality between genders in an asymmetric equilibrium, but overall societal income will also be lower when society is stuck in an asymmetric equilibrium with gender inequality versus the symmetric equilibrium with gender equality.[9]

5 Conclusion

The model presented above provides a new theoretical mechanism that can potentially account for several important empirical patterns regarding differences

[9] There is also a similarity between this result and what arises in Burdett and Coles (2001). In particular, their model also allows for multiple equilibria in the marriage market, with one equilibrium having greater investment by both sides. However, their model is quite distinct from the one developed here and Nosaka's model in the sense that their investment is in a characteristic *only* valued in the marriage market (beauty), and moreover, all of their equilibria are symmetric across genders.

across genders. Most obviously, the model provides a strategic reason behind the social norm of men caring more about the beauty of their potential mates and women caring more about the earnings of their potential mates, that does not require males and females to differ substantially (or even at all) in their underlying preferences, abilities, or opportunities. Maybe even more importantly, the model also shows that this social norm regarding marriage market preferences may be fundamentally tied to gender inequality in the labor market. In particular, gender differences in expectations regarding how different characteristics will be valued in the marriage market may affect individuals' decisions regarding human capital investment. Moreover, the historically widespread limitations against females in schooling and the labor market suggest that in an asymmetric equilibrium of the type described by this model, males would likely be the higher-earning gender prioritizing beauty in the marriage market, while females would be the lower-earning gender that more often prioritizes earnings in a spouse, consistent with the empirical evidence discussed at the outset of the introduction.

While this model emphasizes a particular type of economic pressure that can contribute to gender labor market inequality and gender differences in priorities over spousal characteristics, this model is not meant to suggest that arguments concerning gender inequality that rely on other types of gender norms or evolutionary biological pressures are necessarily incorrect. Rather, the model presented here is simply meant to show how economic pressures may exacerbate and perpetuate other social and biological pressures that lead to inequality across genders. For example, given high-earning males match with high-beauty females, it is easy to see why matching with a high-beauty female may confer a type of observable status to males, causing even low-earning males to desire to match with a high-beauty spouse over a high-earner. Similarly, given older males (i.e. those who have already realized their earnings outcomes) tend to strongly value beauty in female mates, younger males (i.e. those who have yet to complete their human capital investments) may choose to imitate these priorities of older males, strengthening this social norm.

In summary, while the model presented above is admittedly an overly simplistic generalization of the real world, it does highlight two arguably quite general issues. First, one gender's expectations of earnings outcomes for the opposite gender may influence its member's expectations regarding how the marriage market will work, which in turn may influence the human capital investment decisions made by members of that gender. Second, this influence of expectations on human capital investment can cause expectations to become self-fulfilling, potentially leading to not only inequality in earnings across genders, but also differences in priorities over spousal characteristics in the marriage market, even when males and females constitute ex-ante identical populations. Moreover, from a more applied perspective, the model also shows how human capital investment may not only be tied to its payoff in the labor market, but also fundamentally tied to societal expectations regarding how the marriage market will work. Therefore, while policies that lessen constraints against women with respect to the labor market and human capital investment opportunities may help reduce gender labor market inequality, such policies may not be able to completely eradicate inequality if they do not also alter individuals' expectations of what will be valued in the marriage market.

6 Appendix

6.1 Proof of Lemma 1

To prove Lemma 1, recall from equation (1) that all individuals with high-earnings will have marital priorities such that

$$\{\overline{w}, b_h\} \succ \{\underline{w}, b_h\} \succ \{\overline{w}, b_\ell\} \succ \{\underline{w}, b_\ell\}$$

if he or she ends up in a high-earning job, and

$$\{\overline{w}, b_h\} \succ \{\overline{w}, b_\ell\} \succ \{\underline{w}, b_h\} \succ \{\underline{w}, b_\ell\}$$

if he or she ends up in a low-earning job.

Now, note that since type-$\{\overline{w}, b_h\}$ individuals are the most preferred type from both genders regardless of earnings outcomes, they will match together to the greatest extent possible. Since $\pi_h^a \geq \pi_h^b$, we know that there can be at most $\pi_h^b N$ of these matches. Therefore, we know $p_h^b(\{\overline{w}, b_h\}|\overline{w}) = 1$, with all other outcomes for this type equal to zero. However, if $\pi_h^a > \pi_h^b$, there will be some type-$\{\overline{w}, b_h\}$ individuals from gender a who will not be able to match with a similar gender b individual. Therefore, these gender a individuals will match with their second most preferred type, which is type-$\{\underline{w}, b_h\}$ individuals, who will accept such proposals. Therefore, there will be $(\pi_h^a - \pi_h^b)N$ such matches.

Given the above reasoning, we know $p_h^a(\{\overline{w}, b_h\}|\overline{w}) = \frac{\pi_h^b}{\pi_h^a}$ and $p_h^a(\{\overline{w}, b_h\}|\overline{w}) = \frac{\pi_h^a - \pi_h^b}{\pi_h^a}$, with all other outcomes for this type equal to zero.

The remainder of type-$\{\underline{w}, b_h\}$ individuals from gender b who do not match with type-$\{\overline{w}, b_h\}$ individuals will then want to match with their next most preferred type, which is type-$\{\overline{w}, b_\ell\}$ individuals from gender a. Such gender a type-$\{\overline{w}, b_\ell\}$ individuals will be willing to form these matches, as they can not hope to do better than matching with a type-$\{\underline{w}, b_h\}$ individual given all of the type-$\{\overline{w}, b_h\}$ individuals from gender b match with type-$\{\overline{w}, b_h\}$ individuals from gender a. Therefore, there are $[(1-\pi_h^b) - (\pi_h^a - \pi_h^b)]N$, or $(1-\pi_h^a)N$ of matches between type-$\{\underline{w}, b_h\}$ individuals from gender b and type-$\{\overline{w}, b_\ell\}$ individuals from gender a. This means $p_h^b(\{\overline{w}, b_h\}|\underline{w}) = \frac{\pi_h^a - \pi_h^b}{1-\pi_h^b}$ and $p_h^b(\{\overline{w}, b_h\}|\underline{w}) = \frac{1-\pi_h^a}{1-\pi_h^b}$, with all other outcomes for this type equal to zero.

Before consider the remaining type-$\{\overline{w}, b_\ell\}$ individuals from gender a who do not match with a type-$\{\underline{w}, b_h\}$ individuals from gender b, let us consider the type-$\{\overline{w}, b_\ell\}$ individuals from gender b. Like everyone, such individuals would most like to match with type-$\{\overline{w}, b_h\}$ individuals from gender a. However, they will not be able to since such individuals will all match with a high beauty individual from gender b as discussed above. The next most preferred type for type-$\{\overline{w}, b_\ell\}$ individuals from gender b is type-$\{\underline{w}, b_h\}$ individuals from gender a. Such individuals from gender a would also most prefer to match with type-$\{\overline{w}, b_h\}$ individuals from gender b, but they will also not be able to do so. Their next most preferred type is type-$\{\overline{w}, b_\ell\}$ individuals from gender b. Hence, type-$\{\overline{w}, b_\ell\}$ individuals from gender b will match with type-$\{\underline{w}, b_h\}$ individuals from gender a to the greatest extent possible. If $\pi_\ell^b \geq 1 - \pi_h^a$, then there will be $(1-\pi_h^a)N$ of these matches. This means $p_h^b(\{\overline{w}, b_\ell\}|\underline{w}) = 1$, with all other outcomes for this type equal to zero.

Now let us consider those type-$\{\overline{w}, b_\ell\}$ individuals from gender a who do not match with a type-$\{\underline{w}, b_h\}$ individuals from gender b and those type-$\{\overline{w}, b_\ell\}$ individuals from gender b who do not match with a type-$\{\underline{w}, b_h\}$ individuals from gender a. Such individuals would prefer to match with each other than with type-$\{\underline{w}, b_\ell\}$ individuals, which are the least preferred type by all. Given there are $\pi_\ell^a - (1-\pi_h^a)$ of such individuals from gender a, and $\pi_\ell^b - (1-\pi_h^a)$ of such individuals from gender b, and $\pi_\ell^a \geq \pi_\ell^b$, we know there can be at most $\pi_\ell^b - (1-\pi_h^a)$ of such matches. This in turn will mean $p_\ell^b(\{\underline{w}, b_h\}|\overline{w}) = \frac{1-\pi_h^a}{\pi_\ell^b}$ and $p_\ell^b(\{\overline{w}, b_\ell\}|\overline{w}) = \frac{\pi_\ell^b - (1-\pi_h^a)}{\pi_\ell^b}$, with all other outcomes for this type equal to zero.

For those type-$\{\overline{w}, b_\ell\}$ individuals from gender a who do not match with a type-$\{\underline{w}, b_h\}$ individuals from gender b or type-$\{\overline{w}, b_\ell\}$ individuals from gender b, will have no other option than to match with with the least preferred type-$\{\underline{w}, b_\ell\}$ individuals from gender b. Therefore, there will be $(\pi_\ell^a - \pi_\ell^b)N$ of these matches. This means that $p_\ell^a(\{\underline{w}, b_h\}|\overline{w}) = \frac{1-\pi_h^a}{\pi_\ell^a}$, $p_\ell^a(\{\overline{w}, b_\ell\}|\overline{w}) = \frac{\pi_\ell^b-(1-\pi_h^a)}{\pi_\ell^a}$, and $p_\ell^a(\{\underline{w}, b_\ell\}|\overline{w}) = \frac{\pi_\ell^a-\pi_\ell^b}{\pi_\ell^a}$, with $p_\ell^a(\{\overline{w}, b_h\}|\overline{w}) = 0$.

Finally, those type-$\{\underline{w}, b_\ell\}$ individuals from gender b who do not match with type-$\{\overline{w}, b_\ell\}$ individuals from gender a will match with the type-$\{\underline{w}, b_\ell\}$ individuals from gender a. There will be $(1 - \pi_\ell^a)N$ of these matches. This means $p_\ell^b(\{\overline{w}, b_\ell\}|\underline{w}) = \frac{\pi_\ell^a-\pi_\ell^b}{1-\pi_\ell^b}$ and $p_\ell^b(\{\overline{w}, b_\ell\}|\overline{w}) = \frac{1-\pi_\ell^a}{1-\pi_\ell^b}$, with all other outcomes for this type equal to zero, and $p_\ell^a(\{\overline{w}, b_\ell\}|\overline{w}) = 1$, with all other outcomes for this type equal to zero. This completes the proof of Lemma 1.

6.2 Proof of Proposition 1

Let us consider an equilibrum $\{\pi_h^a = 1, \pi_\ell^a = 1, \pi_h^b = 0, \pi_\ell^b = 0\}$, which implicitly an equilibrium where all members of gender a invest in human capital, no members of gender b invest in human capital and matching beliefs are given by plugging $\{\pi_h^a = 1, \pi_\ell^a = 1, \pi_h^b = 0, \pi_\ell^b = 0\}$ into Lemma 1. Given $\Delta v(\overline{w}) < c < \Delta v(\underline{w})$, we can confirm this is in equilibrium by seeing if an individual of any type has an incentive to deviate given equilibrium consistent beliefs.

Gender-a/type-b_h individuals: If invest, then from Lemma 1 $p_h^a(\{\underline{w}, b_h\}|\overline{w}) = 1$ and all other matching probabilities equal zero. If no invest, then from Lemma 1 $p_h^a(\{\underline{w}, b_\ell\}|\underline{w}) = 1$ and all other matching probabilites equal zero. Therefore, from equation (2) a gender-a/type-b_h individual should invest in human capital if and only if $[v(\overline{w} + \underline{w}) + b_h] - c > [v(\underline{w} + \underline{w}) + b_\ell]$, or only if $\Delta v(\underline{w}) + \Delta b > c$, which obviously holds given the condition that $\Delta v(\overline{w}) < c < \Delta v(\underline{w})$. Therefore, individuals of this type have no incentive to deviate from the equilibrium strategy of investing in human capital.

Gender-a/type-b_ℓ individuals: If invest, then from Lemma 1 $p_\ell^a(\{\underline{w}, b_\ell\}|\overline{w}) = 1$ and all other matching probabilities equal zero. If no invest, then from Lemma 1 $p_\ell^a(\{\underline{w}, b_\ell\}|\underline{w}) = 1$ and all other matching probabilites equal zero. Therefore, from equation (2) a gender-a/type-b_ℓ individual should invest in human capital if and only if $[v(\overline{w} + \underline{w}) + b_\ell] - c > [v(\underline{w} + \underline{w}) + b_\ell]$, or only if $\Delta v(\underline{w}) > c$, which obviously holds given the condition $\Delta v(\overline{w}) < c < \Delta v(\underline{w})$.Therefore, individuals of this type have no incentive to deviate from the equilibrium strategy of investing in human capital.

Gender-b/type-b_h individuals: If invest, then from Lemma 1 $p_h^b(\{\overline{w}, b_h\}|\overline{w}) = 1$ and all other matching probabilities equal zero. If no invest, then from Lemma 1 $p_h^b(\{\overline{w}, b_h\}|\underline{w}) = 1$ and all other matching probabilites equal zero. Therefore, from equation (2) a gender-b/type-b_h individual should invest in human capital if and only if $[v(\overline{w}+\overline{w}) + b_h] - c > [v(\overline{w}+\underline{w}) + b_h]$, or only if $\Delta v(\overline{w}) > c$, which obviously cannot hold given the condition $\Delta v(\overline{w}) < c < \Delta v(\underline{w})$.Therefore, individuals of this type have no incentive to deviate from the equilibrium strategy of not investing in human capital.

Gender-b/type-b_ℓ individuals: If invest, then from Lemma 1 $p_\ell^b(\{\overline{w}, b_\ell\}|\overline{w}) = 1$ and all other matching probabilities equal zero. If no invest, then from Lemma 1 $p_\ell^b(\{\overline{w}, b_\ell\}|\underline{w}) = 1$ and all other matching probabilites equal zero. Therefore, from equation (2) a gender-b/type-b_ℓ individual should invest in human capital if and only if $[v(\overline{w}+\overline{w}) + b_\ell] - c > [v(\overline{w}+\underline{w}) + b_\ell]$, or only if $\Delta v(\underline{w}) > c$, which obviously cannot hold given the condition $\Delta v(\overline{w}) < c < \Delta v(\underline{w})$.Therefore, individuals of this type have no incentive to deviate from the equilibrium strategy of not investing in human capital.

This proves no individual has an incentive to deviate from the proposed Nash Equilibrium strategy given each individuals beliefs regarding matching are consistent with the true match probabilities given everyone else follows their strategy under the proposed equilibrium.

6.3 Proof of Proposition 2

From Lemma 1 and the analysis above, we know that an equilibrium consists of a four-tuple $\{\pi_h^a, \pi_\ell^a, \pi_h^b, \pi_\ell^b\}$ that simultaneously solves equations (5)-(8). To show that there exists a symmetric equilibrium four-tuple, where $\pi_h^a = \pi_h^b$ and $\pi_\ell^a = \pi_\ell^b$, let us define π_h^{sym} as the value that solves $c'(\pi_h^{sym}) = \Delta v(\overline{w}) + \Delta b$ and π_ℓ^{sym} as the value that solves $c'(\pi_h^{sym}) = \Delta v(\underline{w}) + \frac{1-\pi_h^{sym}}{\pi_h^{sym}}(\Delta b - \Delta v(\overline{w})) + \Delta v(\overline{w})$. Now consider a four-tuple $\{\pi_h^{a,sym}, \pi_\ell^{a,sym}, \pi_h^{b,sym}, \pi_\ell^{b,sym}\}$ such that $\pi_h^{a,sym} = \pi_h^{b,sym} = \pi_h^{sym}$ and $\pi_\ell^{a,sym} = \pi_\ell^{b,sym} = \pi_\ell^{sym}$. It is straightforward to confirm that this four-tuple simultaneously solves equations (5)-(8), confirming it is indeed an equilibrium. Moreover, it is symmetric across genders.

6.4 Proof of Proposition 3

From Lemma 1 and the analysis above, we know that an equilibrium consists of a four-tuple $\{\pi_h^a, \pi_\ell^a, \pi_h^b, \pi_\ell^b\}$ that simultaneously solves equations (5)-(8). To show that there exists an asymmetric equilibrium four-tuple, where $\pi_h^a > \pi_h^b$ and $\pi_\ell^a \geq \pi_\ell^b$, let us first consider the following Lemma.

Lemma 2 *Given any $\pi_h^b \in [0, \pi^*)$ (where π^* was defined in the text above), it is optimal for high-beauty gender a individuals to choose a $\pi_h^a = \pi_h(\pi_h^b)$, where $\pi_h(0) = \pi^*$ and $\pi_h(\pi_h^b) > \pi^*$ for all $\pi_h^b \in (0, \pi^*)$.*

Proof. Recall that in an equilibrium where $\pi_h^a \geq \pi_h^b$, optimal behavior for high-beauty gender a individuals will consist of a π_h^a that solves the first order condition $c'(\pi_h^a) = \frac{\pi_h^b}{\pi_h^a}\Delta v(\overline{w}) + \Delta b$ (i.e. equation (5)). Recalling that π^* was defined to be such that $c'(\pi^*) = \Delta b$, we know that $c'(\pi^*) < \frac{\pi_h^b}{\pi^*}\Delta v(\overline{w}) + \Delta b$ for any $\pi_h^b > 0$. Moreover, we know that as π_h^a goes to one, the left-hand side of equation (5) goes to infinity, while the right-hand side of equation (5) goes to $\pi_h^b \Delta v(\overline{w}) + \Delta b$ (a finite number for all $\pi_h^b < 1$). Therefore, by the intermediate value theorem, we know that there exists some $\pi_h(\pi_h^b)$, such that $\pi_h(\pi_h^b)$ solves equation (5) and $\pi_h(\pi_h^b) > \pi^*$ for any given $\pi_h^b \in (0, \pi^*)$. Moreover, it can easily be confirmed that at $\pi_h^b = 0$, the π_h^a that solves equation (5) will equal π^*, implying $\pi_h(0) = \pi^*$. ■

Now recall that in an equilibrium where $\pi_h^a \geq \pi_h^b$, optimal behavior for high-beauty gender b individuals is to choose a π_h^b that solves the first order condition $c'(\pi_h^b) = \Delta v(\overline{w}) + \frac{1-\pi_h^a}{1-\pi_h^b}\Delta b$ (i.e. equation (6)). From the Lemma above, we know that in such an equilibrium it also must be the case that high-beauty gender a individuals choose a $\pi_h^a = \pi_h(\pi_h^b)$, where $\pi_h(\pi_h^b) \geq \pi^*$ for any given $\pi_h^b \in [0, \pi^*)$. Substituting this into equation (6), we can see that in equilibrium where $\pi_h^a \geq \pi^* \geq \pi_h^b$, the following equation must hold

$$c'(\pi_h^b) = \Delta v(\overline{w}) + \frac{1 - \pi_h(\pi_h^b)}{1 - \pi_h^b}\Delta b. \tag{9}$$

Now, note that if $\pi_h^b = 0$, the left-hand side of equation (9) equals zero while the right-hand side is strictly positive (given $\pi_h(0) = \pi^* > 0$), meaning the left-hand side of equation (9) is less than the right-hand side at $\pi_h^b = 0$. Next, note that if there exists a $\pi < \pi^*$ such that $c'(\pi) \geq \Delta v(\overline{w}) + \frac{1-\pi^*}{1-\pi}\Delta b$, and recalling that $\pi_h(\pi_h^b) > \pi^*$ for all $\pi_h^b < \pi^*$, then if we know there exists $\pi < \pi^*$ such that $c'(\pi) > \Delta v(\overline{w}) + \frac{1-\pi_h(\pi)}{1-\pi}\Delta b$, or where the left-hand side of equation (9) exceeds the right-hand side for some $\pi < \pi^*$. Therefore, since both sides of equation (9) are continuous, we then can apply the intermediate value theorem to confirm that there exists a π_h^b that solves equation (9), which we can denote $\pi_h^{b,asym}$, and is such that $\pi_h^{b,asym} < \pi^*$. Given this equilibrium value for $\pi_h^{b,asym}$, we can determine the equilibrium π_h^a by setting it equal to $\pi_h(\pi_h^{b,asym})$ and denote this value as $\pi_h^{a,asym}$. Moreover, given $\pi_h^{b,asym} < \pi^*$ and the fact that $\pi_h(\pi_h^b) > \pi^*$ for all $\pi_h^b \in [0, \pi^*)$, we know that $\pi_h^{a,asym} > \pi^* > \pi_h^{b,asym}$ in such an equilibrium. Moreover, plugging $\pi_h^{a,asym}$ into equations (7) and (8), it is easy to confirm that there exists a π_ℓ that solves equations (7) and (8) when $\pi_\ell^a = \pi_\ell^a = \pi_\ell$. This confirms part one of Proposition 2.

In order to prove part 2 of Proposition 2, note that if there is $\pi < \pi^*$ such that $c'(\pi) \geq \Delta v(\overline{w}) + \frac{1-\pi^*}{1-\pi}\Delta v(\underline{w}) + \frac{1-\pi^*}{\pi^*}(\Delta b - \Delta v(\overline{w}))$, then since $v(\underline{w}) > \Delta b$, it will also be true that there is $\pi < \pi^*$ such that $c'(\pi) > \Delta v(\overline{w}) + \frac{1-\pi^*}{1-\pi}\Delta b$. Therefore, from above that if there exists a $\pi < \pi^*$ such that $c'(\pi) \geq \Delta v(\overline{w}) + \frac{1-\pi^*}{1-\pi}\Delta v(\underline{w}) + \frac{1-\pi^*}{\pi^*}(\Delta b - \Delta v(\overline{w}))$, we know that there exists a pair $\{\pi_h^{a,asym}, \pi_h^{b,asym}\}$ that solve equations (5) and (6) and are such that $\pi_h^{a,asym} > \pi^* > \pi_h^{b,asym}$. Given this, we can prove the remainder of part 2 of Proposition 2 in much the same manner as we proved part 1. In particular, consider the following Lemma.

Lemma 3 *Given any* $\pi_\ell^b \in [0, \pi^*)$ *(where* π^* *was defined in the text above) and* $\pi_h^a = \pi_h^{a,asym}$ *(where* $\pi_h^{a,asym}$ *is defined above in the proof of part 1 of this Proposition), it is optimal for low-beauty gender a individuals to choose a* $\pi_\ell^a = \pi(\pi_\ell^b)$*, where* $\pi_\ell(\pi_\ell^b) \geq \pi^*$ *for all* $\pi_\ell^b \in [0, \pi^*)$ *and* $\pi_\ell(0) > \pi^*$*.*

Proof. First recall that optimal behavior for low-beauty gender a individuals given $\pi_h^a = \pi_h^{a,asym}$ is to choose a π_ℓ^a that solves the first order condition $c'(\pi_\ell^a) = \Delta v(\underline{w}) + \frac{1-\pi_h^{a,asym}}{\pi_\ell^a}\Delta b + \frac{\pi_\ell^b}{\pi_\ell^a}\Delta v(\overline{w})$ (i.e. equation (7)). Noting since π^* was defined to be such that $c'(\pi^*) = \Delta b$, we know $c'(\pi^*) < \Delta v(\underline{w}) + \frac{1-\pi_h^{a,asym}}{\pi^*}\Delta b + \frac{\pi_\ell^b}{\pi^*}\Delta v(\overline{w})$ for any $\pi_\ell^b > 0$, meaning that the left-hand side of equation (7) is less than the right-hand side when $\pi_\ell^a = \pi^*$ for any $\pi_\ell^b > 0$. Moreover, we know that as π_ℓ^a goes to one, the left-hand side of equation (7) goes to infinity, while the right-hand side of equation (7) goes to $\Delta v(\underline{w}) + (1 - \pi_h^{a,asym})\Delta b + \pi_\ell^b \Delta v(\overline{w})$ (which is finite for all $\pi_\ell^b \leq 1$). Therefore, by the intermediate value theorem, we know that there exists some $\pi_\ell^a = \pi_\ell(\pi_\ell^b)$ such that $\pi_\ell(\pi_\ell^b)$ solves equation (7) for any given $\pi_\ell^b \in [0, \pi^*)$ and $\pi^* < \pi_\ell(\pi_\ell^b) < 1$. Moreover, it can easily be confirmed that $\pi(0) > \pi^*$. ■

Finally, recall that optimal behavior for low-beauty gender b individuals is to choose a π_ℓ^b that solves the first order condition $c'(\pi_\ell^b) = \Delta v(\overline{w}) + \frac{1-\pi_\ell^a}{1-\pi_\ell^b}\Delta v(\underline{w}) + \frac{1-\pi_h^a}{\pi_\ell^a}\Delta b$ (i.e. equation (8)). From above, we know that in an equilibrium where $\pi_h^a = \pi_h^{a,asym}$, low-beauty gender a individuals will choose a $\pi_\ell^a = \pi_\ell(\pi_\ell^b)$, where $\pi_\ell(\pi_\ell^b) > \pi^*$, for any given $\pi_\ell^b \in [0, \pi^*)$. Substituting this result into equation (8), we can see that in an equilibrium where $\pi_\ell^a > \pi^* > \pi_\ell^b$, the following equation must hold

$$c'(\pi_\ell^b) = \Delta v(\overline{w}) + \frac{1 - \pi_\ell(\pi_\ell^b)}{1 - \pi_\ell^b}\Delta v(\underline{w}) + \frac{1 - \pi_h^{a,asym}}{\pi_\ell(\pi_\ell^b)}(\Delta b - \Delta v(\overline{w})). \qquad (10)$$

Now note that if $\pi_\ell^b = 0$, the left-hand side of equation (10) equals zero while the right-hand side is strictly positive, meaning that at $\pi_\ell^b = 0$ the right-hand side of equation (10) is less than the left-hand side. Next, note that if there exists a $\pi < \pi^*$ such that $c'(\pi) \geq \Delta v(\overline{w}) + \frac{1-\pi^*}{1-\pi}v(\underline{w}) + \frac{1-\pi^*}{\pi^*}(\Delta b - \Delta v(\overline{w}))$, then we know that since $\pi_h^{a,asym} > \pi^*$ and $\pi_\ell(\pi_\ell^b) > \pi^*$ for all $\pi_\ell^b \in (0, \pi^*)$, it will also be true that there exists a $\pi < \pi^*$ such that $c'(\pi) > \Delta v(\overline{w}) + \frac{1-\pi_\ell(\pi_\ell^b)}{1-\pi}v(\underline{w}) + \frac{1-\pi_h^{a,asym}}{\pi_\ell(\pi_\ell^b)}(\Delta b - \Delta v(\overline{w}))$, meaning that for some $\pi < \pi^*$ the right-hand side of equation (10) exceeds the left-hand side. Therefore, since both sides of equation (10) are continuous, we then can apply the intermediate value theorem to confirm that there exists a π_ℓ^b that solves equation (9), which we can denote $\pi_\ell^{b,asym}$, and is such that $\pi_\ell^{b,asym} < \pi^*$. Given this equilibrium value for $\pi_\ell^{b,asym}$, we can determine the equilibrium π_ℓ^a by setting it equal to $\pi_\ell(\pi_\ell^{b,asym})$ and denote this value as $\pi_\ell^{a,asym}$. Moreover, given $\pi_\ell^{b,asym} < \pi^*$ and the fact that $\pi_\ell(\pi_\ell^b) \geq \pi^*$ for all $\pi_\ell^b \in [0, \pi^*)$, we know that $\pi_\ell^{a,asym} > \pi^* > \pi_\ell^{b,asym}$ in such an equilibrium. This confirms part two of Proposition 2.

6.5 Proof of Proposition 4

To prove Proposition 4, first note that in the symmetric equilibrium it will be true that $\pi_h^a = \pi_h^b = \pi_h^{sym}$, where π_h^{sym} solves

$$c'(\pi_h^{sym}) = \Delta v(\overline{w}) + \Delta b. \qquad (11)$$

Alternatively, in an asymmetric equilibrium, $\pi_h^a = \pi_h^{a,asym}$ and $\pi_h^b = \pi_h^{b,asym}$, where $\pi_h^{a,asym} > \pi_h^{b,asym}$ and

$$c'(\pi_h^{a,asym}) = \frac{\pi_h^{b,asym}}{\pi_h^{a,asym}} \Delta v(\overline{w}) + \Delta b \tag{12}$$

and

$$c'(\pi_h^{a,asym}) = \Delta v(\overline{w}) + \frac{1 - \pi_h^{a,asym}}{1 - \pi_h^{b,asym}} \Delta b. \tag{13}$$

Given $\pi_h^{a,asym} > \pi_h^{b,asym}$, simple inspection of the above three equations shows that the right-hand side of equation (11) exceeds the right-hand sides of both equation (12) and equation (13). Therefore, $\pi_h^{sym} > \pi_h^{a,asym} > \pi_h^{b,asym}$, meaning high-beauty individuals of both genders have higher average earnings in the symmetric equilibrium than an asymmetric equilibrium.

Now, let us further consider an environment where there exists an asymmetric equilibrium where not only does $\pi_h^a > \pi_h^b$, but it is also the case that $\pi_\ell^a > \pi_\ell^b$. Using an identical argument to the previous paragraph, we know that in such an equilibrium, $\pi_h^a = \pi_h^{a,asym}$ and $\pi_h^b = \pi_h^{b,asym}$, where $\pi_h^{sym} > \pi_h^{a,asym} > \pi_h^{b,asym}$. Furthermore, we also know that in such and equilibrium it will be true that $\pi_\ell^a = \pi_\ell^{a,asym}$ and $\pi_\ell^b = \pi_\ell^{b,asym}$, where $\pi_\ell^{a,asym} > \pi_\ell^{b,asym}$ and

$$c'(\pi_\ell^{a,asym}) = \Delta v(\underline{w}) + \frac{1 - \pi_h^{a,asym}}{\pi_\ell^{a,asym}} (\Delta b - \Delta v(\overline{w})) + \frac{\pi_\ell^{b,asym}}{\pi_\ell^{a,asym}} \Delta v(\overline{w}) \tag{14}$$

and

$$c'(\pi_\ell^{b,asym}) = \frac{1 - \pi_\ell^{a,asym}}{1 - \pi_\ell^{b,asym}} \Delta v(\underline{w}) + \frac{1 - \pi_h^{a,asym}}{\pi_\ell^{b,asym}} (\Delta b - \Delta v(\overline{w})) + \Delta v(\overline{w}). \tag{15}$$

Alternatively, in the symmetric equilibrium, it will be true that $\pi_h^a = \pi_h^a = \pi_h^{sym}$ (as defined above) and $\pi_\ell^a = \pi_\ell^b = \pi_\ell^{sym}$, where

$$c'(\pi_\ell^{sym}) = \Delta v(\underline{w}) + \frac{1 - \pi_h^{sym}}{\pi_\ell^{sym}} (\Delta b - \Delta v(\overline{w})) + \Delta v(\overline{w}). \tag{16}$$

Once again, simple inspection of the above three equations reveals that when $\pi_\ell^{a,asym} > \pi_\ell^{b,asym}$, the right-hand side of equation (16) exceeds the right-hand sides of equations (14) and (15), implying $\pi_\ell^{sym} > \pi_\ell^{a,asym} > \pi_\ell^{b,asym}$, meaning low-beauty individuals will also have lower average earnings than they would in the symmetric equilibrium.

References

[1] Albanesi, Stefania and Claudia Olivetti. (2005). "Home Production, Market Production and the Gender Wage Gap: Incentives and Expectations." CEPR Discussion Paper 4984.

[2] Becker, Gary S. (1973). "A Theory of Marriage : Part I." Journal of Political Economy 81: 813-46.

[3] —. (1974). "A Theory of Marriage: Part II." Journal of Political Economy 82(2, part 2): S11-S26.

[4] Bergstrom, Theodore and Mark Bagnoli. (1993). "Courtship as a Waiting Game." Journal of Political Economy 101(1): 185-202.

[5] Biddle, Jeff E. and Daniel Hammermesh. (1998). "Beauty, Productivity, and Discrimination: Lawyers' Looks and Lucre." Journal of Labor Economics 16(1): 172-201.

[6] Bjerk, David and Seungjin Han. (2007). "Assortative Marriage and the Effects of Government Homecare Subsidy Programs on Gender Wage and Participation Inequality." Journal of Public Economics 91(5-6): 1135-1150.

[7] Burdett, Kenneth and Melvyn Coles. (1997). "Marriage and Class." Quarterly Journal of Economics 112: 141-168.

[8] ——. (1999). "Long-Term Partnership Formation: Marriage and Employment." Economic Journal 109: F307-F334.

[9] ——. (2001). "Transplants and Implants." International Economic Review 42: 597-616.

[10] Buss, David M. (1987). "Sex Differences in Human Mate Selection Criteria: An Evolutionary Perspective." In *Sociobiology and Psychology: Ideas, Issues, and Applications*. Hillsdale, NJ: Erlbaum.

[11] ——. (1989). "Sex Differences in Human Mate Preferences: Evolutionary Hypotheses Tested in 37 Cultures." Behavioral and Brain Sciences 12: 1-49.

[12] ——. (1994). The Evolution of Desire: Strategies of Human Mating. New York, NY: Basic Books, p. 58.

[13] Cole, Harold Linh, George Mailath, and Andrew Postlewaite. (2001). "Efficient Non-Contractible Investments in Finite Economies." B.E. Press Advances in Theoretical Economics 1: 1-32.

[14] Fernandez, Raquel, Nezih Guner, and John Knowles. (2005). "Love and Money: A Theoretical and Empirical Analysis of Household Sorting and Inequality." Quarterly Journal of Economics 120: 273-344.

[15] Fisman, Raymond, Sheena S. Iyengar, Emir Kamenica, Itamar Simonson. (2006). "Gender Differences in Mate Selection: Evidence from a Speed Dating Experiment." Quarterly Journal of Economics 121(2): 673-697.

[16] Francois, Patrick. (1998). "Gender Discrimination Without Gender Difference: Theory and Policy Responses." Journal of Public Economics 68: 1-32.

[17] Gale, David and Lloyd S. Shapley. (1962). "College Admissions and the Stability of Marriages." American Mathematical Monthly 69: 9-15.

[18] Hammermesh, Daniel and Jeff Biddle. (1994). "Beauty and the Labor Market." American Economic Review 84(5): 1174-94.

[19] Hess, Gregory D. (2004). "Marriage and Consumption Insurance: What's Love Got to Do With It?" Journal of Political Economy 112: 290-318.

[20] Hill, R. (1945). "Campus Values and Mate Selection." Journal of Home Economics 37: 554-558.

[21] Hitsch, Gunter, Ali Hortacsu, and Dan Ariely. (2005). "What Makes You Click: An Empirical Analysis of Online Dating." Working Paper, University of Chicago Department of Economics.

[22] Hudson, J.W. and L.F. Henze. (1969). "Campus Values and Mate Selection: A Replication." Journal of Marriage and the Family 31: 772-775.

[23] Kotlikoff, L. and A. Spivak. (1981). "The Family as an Incomplete Annuity Market." Journal of Political Economy 89: 372-391.

[24] Lam, David. (1988). "Marriage Markets and Assortative Mating with Household Public Goods: Theoretical Results and Empirical Implications." Journal of Human Resources 23: 462-487.

[25] Lazear, Edward and Sherwin Rosen. (1990). "Male-Female Wage Differentials in Job Ladders." Journal of Labor Economics 8: S106-S123.

[26] Lommerud, Kjell Erik and Steinar Vagstad. (2000). "Mommy Tracks and Public Policy: On Self-Fulfilling Prophecies and Gender Gaps in Promotion." CEPR Discussion Paper 2378.

[27] Mailath, George J. and Andrew Postlewaite. (2004). "Social Assets." PIER Working Paper 04-025.

[28] McGinnis R. (1958). "Campus Values in Mate Selection." Social Forces 35: 368-373.

[29] Mobius, Mark and Tanya Rosenblat. (2006). "Why Beauty Matters." American Economic Review 96(1): 222-235.

[30] Mortensen, Dale. (1988). "Matching: Finding a Partner for Life or Otherwise." American Journal of Sociology 94: S215-S240.

[31] Nosaka, Hiromi. (2007). "Specialization and Competition in Marriage Models." Journal of Economic Behavior and Organization 63:104-119.

[32] Ogaki, M. and Q. Zhang. (2001). "Decreasing Relative Risk Aversion and Tests of Risk Sharing." Econometrica 69: 515-534.

[33] Peters, Michael. (2005). "The Pre-Marital Investment Game." Working Paper, University of British Columbia Department of Economics.

[34] Peters, Michael and Aloysius Siow. (2002). "Competing Pre-Marital Investments." Journal of Political Economy 110: 592-609.

[35] Rogerson, Richard, Robert Shimer, and Randall Wright. (2005). "Search-Theoretic Models of the Labor Market: A Survey." Journal of Economic Literature 43(4): 959-988.

[36] Rosenzweig, M. and O. Stark. (1989). Consumption Smoothing, Migration, and Marriage: Evidence from Rural India." Journal of Political Economy 97: 905-926.

[37] Roth, Alvin. (1984). "Misrepresentation and Stability in the Marriage Problem." Journal of Economic Theory 34: 383-387.

[38] Roth, Alvin and M.A.O. Sotomayor. (1990). *Two Sided Matching: A Study of Game Theoretic Modeling and Analyis*. New York, NY: Cambridge University Press. Econometric Society Monograph No. 18.

[39] Roth, Alvin and John H. Vande Vate. (1990). "Random Paths to Stability in Two-Sided Matching." Econometrica 58(6): 1475-1480.

[40] Shimer, Robert and Lones Smith. (2000). "Assortative Matching and Search." Econometrica 68: 342-369.

[41] Smith, Lones. (2002). "The Marriage Model with Search Frictions." Working Paper, University of Michigan Department of Economics.