Household Formation and Gender Human Capital Inequality

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Abstract

I consider a model of human capital investment where males and females invest in human capital and then are randomly paired with a member of the opposite gender to form a household, where total earnings are split evenly within the household. The key assumptions of the model are that individuals incur diminishing marginal utility in money, and that some fraction of females are more likely to drop out of the labor market after forming a household than males. In this environment, all females invest less in human capital than males, even those females who are certain they will continue working after forming a household. The intuition for this result is quite simple—males realize there is some chance they will be the sole breadwinner in the household, while even females who plan to work after forming a household know that it is unlikely they will be the sole breadwinner in their households. Therefore, the marginal benefit to additional human capital investment is always greater for males than females.

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1 Introduction

Gender differences in choice of fields of study have been well documented. A variety of papers have shown that female college students are significantly less likely to major in subjects such as Business, Computer Science, Engineering, Economics, and Mathematics than male college students, while being significantly more likely to major in Education, English, and other humanities (Daymont and Andrisani, 1984; Turner and Bowen, 1999; Weinberger, 1999). While gender differences in field of study is not necessarily a reason for concern, the existing discrepancy is notable since the fields of study dominated by males generally lead to significantly higher pay than the fields of study dominated by females (Daymont and Andrisani, 1984). Given these higher labor market returns associated with certain fields of study than others, one way to describe the gender differences in fields of study is to say that females invest less in *labor market human capital* than males, by which I mean human capital that pays off in terms of higher monetary labor market earnings.

Why there exists this significant gender difference in labor market human capital investment remains unresolved. In a recent controversial talk at the National Bureau of Economic Research, Harvard President Lawrence Summers raised the question of whether such differences in human capital investment may be due to "innate" differences between males and females, or whether there was an alternative theory that could plausibly explain this phenonenon (Hemel, 2005).

This paper proposes a very simple model of the labor market and household formation that provides an alternative explanation for gender differences in labor market human capital investment. In the model I develop below, the differences in human capital investment across genders do not arise due to exogenous gender differences in efficiency of human capital production, or even discrimination in the labor market. Indeed, males and females are assumed to have the same costs to producing human capital (i.e. wage enhancing skill) and are treated identically by employers.[1] Rather, the gender human capital inequality that arises in this model comes from the assumptions that males and females form households where earnings are shared relatively equally, utility of each individual exhibits diminishing marginal returns in his/her share of household earnings, and some fraction of females have a greater likelihood of leaving the labor market after forming a household than males.

While it is not surprising that these females who have a higher likelihood of leaving the labor market after forming a household invest less in human capital than males, the more interesting result is that even those females who plan to stay in the labor market after forming a household invest less in human capital than their otherwise identical male counterparts.[2] Intuitively, males know that

[1] The absence of gender discrimination by employers distinguishes this model from other models of gender wage inequality such as Lazear and Rosen (1990) and Francois (1998).

[2] This result distinguishes this model from Becker (1981, 1985), where human captial investment inequality will not necessarily occur between males and females who both plan to stay in the labor market after forming their households.

there is some probability that they will be the sole earner in their households, which makes any earnings associated with additional human capital very beneficial. Alternatively, even females who plan to work after forming a household know that they are unlikely to be the sole earner in their households. Therefore, the benefit coming from any earnings associated with additional human capital investment is relatively smaller for females than it is for males, even for those females who are certain they will remain in the labor force. In this way, this model shows why males may invest more in human capital than females, and subsequently earn more in the labor market than their ex-ante identical female counterparts.

2 Model

In the economy, assume there exist males and females. Both males and females first decide how much to invest in human capital, denoted by e. After making this investment, males and females are then randomly matched to form households. Once in a household, all earnings of household members are shared equally between the two household members. The following sections examine how human capital investment may differ across males and females under slightly different assumptions concerning the types of females in the economy and the payoffs to human capital.

2.1 Simplest Environment

In this simplest environment, assume an individual's earnings are an increasing strictly concave function of the human capital investment. For notational purposes, let $\omega(e)$ denote the earnings for an individual who invested e in human capital, where $\omega'(e) > 0$ and $\omega''(e) < 0$.[3] Furthermore, say there exist two types of females in the economy. Type-1 females are assumed to be identical to males in all respects, while type-2 females differ in that they will choose to leave the labor market after forming a household. Let $\pi \in (0, 1)$ represent the fraction of females that are of type-1.[4] Finally, let each individual's utility from his or her share of houshold earnings ω be given by a utility function $U(\omega)$, where $U'(\omega) > 0$ and $U''(\omega) < 0$, and let the utility cost to investing in human capital simply equal e.

Given this environment, the expected utility for a male is given by

$$V_m(e) = \pi U\left(\frac{\omega(e) + \omega(\widehat{e}_{f1})}{2}\right) + (1 - \pi)U\left(\frac{\omega(e)}{2}\right) - e, \tag{1}$$

[3] Also, assume that $\omega'(0)$ is sufficiently large so that it always makes sense for an individual who plans on working to invest in at least a little bit in human capital.

[4] One could argue that there also could be type-2 males, or males who plan to drop out of the labor market upon forming a household. However, as long as the fraction of males that plan to drop out of the labor market upon household formation is strictly less than the fraction of females who plan to drop out of the labor market upon household formation, all of the results in this section hold. Therefore, for simplicity, I implicitly assume there only exist type-1 males.

where $\widehat{e}_{f1}$ is the expected human capital investment made by a type-1 female household member.[5] Alternatively, the expected utility for type-1 females is given by

$$V_{f1}(e) = U\left(\frac{\omega(\widehat{e}_m) + \omega(e)}{2}\right) - e, \tag{2}$$

where $\widehat{e}_m$ is the expected human capital investment made by the male household member. Similarly, the expected utility for type-2 females is given by

$$V_{f2}(e) = U\left(\frac{\omega(\widehat{e}_m)}{2}\right) - e. \tag{3}$$

Not surprisingly, equation (3) reveals that type-2 females should not invest at all in human capital regardless of what they expect from males, since they obtain no benefit from such an investment since they do not plan on working. We can denote this optimal human capital investment by type-2 females as e^*_{f2}. To find optimal human capital investment for males and type-1 females we can take first order conditions of equations (1) and (2) and set them equal to zero. Doing so with respect to equation (1) we get

$$\frac{\partial V_m(e^*_m)}{\partial e} = \pi U'\left(\frac{\omega(e^*_m) + \omega(\widehat{e}_{f1})}{2}\right)\frac{\omega'(e^*_m)}{2} + (1-\pi)U'\left(\frac{\omega(e^*_m)}{2}\right)\frac{\omega'(e^*_m)}{2} - 1 = 0.$$

Re-arranging the above expression we get

$$\omega'(e^*_m) = \frac{2}{\pi U'\left(\frac{\omega(e^*_m)+\omega(\widehat{e}_{f1})}{2}\right) + (1-\pi)U'\left(\frac{\omega(e^*_m)}{2}\right)}. \tag{4}$$

Taking first order condition of equation (2) and setting it equal to zero we obtain

$$\frac{\partial V_f(e^*_{f1})}{\partial e} = U'\left(\frac{\omega(\widehat{e}_m) + \omega(e^*_{f1})}{2}\right)\frac{\omega'(e^*_{f1})}{2} - 1 = 0,$$

which can be re-arranged to get

$$\omega'(e^*_{f1}) = \frac{2}{U'\left(\frac{\omega(\widehat{e}_m)+\omega(e^*_{f1})}{2}\right)}. \tag{5}$$

Clearly, in equilbrium, it is required that expectations correspond to the truth. Therefore, equilibrium investment in human capital by males and type-1 females is given by a pair $\{e^*_m, e^*_{f1}\}$ that simultaneously solve equations (4) and (5) when $\widehat{e}_m = e^*_m$ and $\widehat{e}_{f1} = e^*_{f1}$. It is straightforward to prove such an

[5] Techincally, expectations should be taken over the whole U function with respect to type-1 female human capital investment. However, since all type-1 females are identical, they will all invest the same amount in human capital, meaning the overall distribution of type-1 female human capital investment degenerates to a single mass point in equilibrium. Hence, it is without loss of generality to write expected utility as given above.

equilibrium pair exists. Moreover, given the concavity of the ω and U functions, we can easily show that $e^*_m > e^*_{f1} > e^*_{f2}$ (see Appendix for formal proof).

In words, this means that in equilibrium, males invest more in human captial than all females, even those females who plan on remaining in the labor force after forming a household. This is interesting given that type-1 females are identical to males in all respects, including the actual labor market return on their human capital investments. Intuitively, the reason males invest more in human capital than type-1 females in equilibrium is because they realize that there is some probability they will form a houshold with a type-2 female (i.e. a female that does not stay in the labor market). When this happens, the male will be the only breadwinner in the household, making the marginal benefit from the earnings gain associated with further human capital investment quite large. On the other hand, type-1 females (i.e. those who plan to stay in the labor market after forming a household) know that they will always have another breadwinner in the household besides them, making the marginal benefit from the earnings gain associated with further human capital investment relatively small.

2.2 Complicating the Model

One concern is that the environment proposed above is overly simplistic and the main result is not robust to realistic complications. In this section I complicate the evironment by making the differences between female types less dramatic and allow for some uncertainty with respect to the payoff to the human capital investment. Specifically, assume that earnings in the labor market can now either be high (denoted ω^h) or low (denoted ω^ℓ). The probability of obtaining a high earnings job in the labor market is assumed to be given by $p(e)$, where e is an individual's investment in human capital and $p(e)$ is an increasing but concave function of e. The corresponding probability of obtaining a low earnings job is given by $1 - p(e)$.

We will continue to assume there exists two types of females in the economy. Once again, let type-1 females be identical in all respects to males, meaning they plan to stay in the labor market after forming a household regardless of their spouses' earnings or their own earnings outcome. However, now assume that type-2 females will continue to work after forming a household only if they end up with a high earnings job. Again, let $\pi \in (0, 1)$ denote the fraction of females who are of type-1.[6]

In this environment, expected utility for males is now given by

[6] Again, males could also be of both types. However, like before, as long as the fraction of males who are of type-2 is smaller than the fraction of females who are of type-2, all the results in this section hold.

$$
\begin{aligned}
V_m(e) = \pi & \left[p(e)p(\widehat{e}_{f1})U\left(\frac{\omega^h+\omega^h}{2}\right) + p(e)(1-p(\widehat{e}_{f1}))U\left(\frac{\omega^h+\omega^\ell}{2}\right) \right. \\
& \left. +(1-p(e))p(\widehat{e}_{f1})U\left(\frac{\omega^\ell+\omega^h}{2}\right) + (1-p(e))(1-p(\widehat{e}_{f1}))U\left(\frac{\omega^\ell+\omega^\ell}{2}\right)\right] \\
& (1-\pi)\left[p(e)p(\widehat{e}_{f2})U\left(\frac{\omega^h+\omega^h}{2}\right) + p(e)(1-p(\widehat{e}_{f2}))U\left(\frac{\omega^h}{2}\right) \right. \\
& \left. (1-p(e))p(\widehat{e}_{f2})U\left(\frac{\omega^\ell+\omega^h}{2}\right) + (1-p(e))(1-p(\widehat{e}_{f2}))U\left(\frac{\omega^\ell}{2}\right)\right] - e
\end{aligned}
$$

where $\widehat{e}_{f1}$ is the expected human captial investment by type-1 females and $\widehat{e}_{f2}$ is the expected human capital investment by type-2 females.[7] Similarly, the expected utility for type-1 females is now given by

$$
\begin{aligned}
V_{f1}(e) = p(e)p(\widehat{e}_m)U\left(\frac{\omega^h+\omega^h}{2}\right) &+ p(e)(1-p(\widehat{e}_m))U\left(\frac{\omega^h+\omega^\ell}{2}\right) \\
+ (1-p(e))p(\widehat{e}_m)U\left(\frac{\omega^\ell+\omega^h}{2}\right) &+ (1-p(e))(1-p(\widehat{e}_m))U\left(\frac{\omega^\ell+\omega^\ell}{2}\right) - e,
\end{aligned}
$$

and the expected utility for type-2 females is now given by

$$
\begin{aligned}
V_{f2}(e) = p(e)p(\widehat{e}_m)U\left(\frac{\omega^h+\omega^h}{2}\right) &+ p(e)(1-p(\widehat{e}_m))U\left(\frac{\omega^h+\omega^\ell}{2}\right) \\
+ (1-p(e))p(\widehat{e}_m)U\left(\frac{\omega^h}{2}\right) &+ (1-p(e))(1-p(\widehat{e}_m))U\left(\frac{\omega^\ell}{2}\right) - e,
\end{aligned}
$$

where $\widehat{e}_m$ is the expected human captial investment by males.

Once again, to find the optimal investment by males and females of each type, we can take the derivative of the above expressions and set them equal to zero. Doing so for males, then re-arranging and simplifying gives

$$
\begin{aligned}
p'(e_m^*) = & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (6) \\
& \left[\pi \left[p(\widehat{e}_{f1})\left(U\left(\omega^h\right) - U\left(\frac{\omega^h+\omega^\ell}{2}\right)\right) + (1-p(\widehat{e}_{f1}))\left(U\left(\frac{\omega^h+\omega^\ell}{2}\right) - U\left(\omega^\ell\right)\right)\right] + \right. \\
& \left. (1-\pi)\left[p(\widehat{e}_{f2})\left(U\left(\omega^h\right) - U\left(\frac{\omega^h+\omega^\ell}{2}\right)\right) + (1-p(\widehat{e}_{f2}))\left(U\left(\frac{\omega^h}{2}\right) - U\left(\frac{\omega^\ell}{2}\right)\right)\right]\right]^{-1}.
\end{aligned}
$$

Doing the same for females we get

[7] Again, expectations should be taken over the whole U function with respect to type-1 and type-2 female human capital investment. However, like before, all females of the same type are identical, meaning they will all invest the same amount in human capital. Hence, writing expected utility in this manner is without consequence.

$$p'(e^*_{f1}) = \tag{7}$$
$$\left[p(\widehat{e}_m)\left(U\left(\omega^h\right) - U\left(\frac{\omega^h+\omega^\ell}{2}\right)\right) + (1-p(\widehat{e}_m))\left(U\left(\frac{\omega^h+\omega^\ell}{2}\right) - U\left(\omega^\ell\right)\right)\right]^{-1},$$

for type-1 females, and

$$p'(e^*_{f2}) = \tag{8}$$
$$\left[p(\widehat{e}_m)\left(U\left(\omega^h\right) - U\left(\frac{\omega^h}{2}\right)\right) + (1-p(\widehat{e}_m))\left(U\left(\frac{\omega^h+\omega^\ell}{2}\right) - U\left(\frac{\omega^\ell}{2}\right)\right)\right]^{-1}$$

for type-2 females.

Once again, in equilibrium it must be the case that expectations correspond to the truth, meaning an equilibrium consists of a triplet $\{e^*_m, e^*_{f1}, e^*_{f2}\}$ that solves equations (6) -(8) when $\widehat{e}_m = e^*_m$, $\widehat{e}_{f1} = e^*_{f1}$, and $\widehat{e}_{f2} = e^*_{f2}$. It is again straightforward to prove existence of such an equilibrium.

Given such an equilibrium, the concavity of the p function and equations (7) and (8) directly imply that $e^*_{f1} > e^*_{f2}$. In other words, females who plan on staying in the labor market regardless of their earnings outcome will invest more in human capital than females who plan to only stay in the labor market if they turn out to land a high earnings job. More notably, it will also be the case that $e^*_m > e^*_{f1}$, meaning males will invest more in human capital than even females who plan to stay in the labor market with certainty (as well as females who only plan on staying in the labor market if they obtain a high earnings job). This assertion is once again proved in the Appendix, but the intution is identical to that in the previous section—males know that with some probabilty they may turn out to be the only breadwinner in their household, while type-1 females know that they will always have another breadwinner in their household. This makes the marginal utility benefit for investing in further human capital always greater for males than for females, even those females who plan on staying in the labor market with certainty.

2.3 Further Complicating the Model

In the model developed above, it was always assumed that there is only one initial skill type of individual, where initial skill type can refer to either how costly human capital investment is, or the magnitude of the earnings payoff associated with any fixed human capital investment. However, note that the model can easily be generalized to multiple initial skill types, as long as initial skill types are known at the time of the human capital investment and males and females form households assortatively by initial skill type.[8] With this extension,

[8] In fact, assortative matching in initial skill is not necessary. As long as the matching process is symmetric for males and females, nothing in the model is affected by having multiple initial skill types.

the model developed above can be directly interpreted as showing how males and females of similar initial skill invest differently in human capital.

Another reasonable complication to the model above is to assume that type-2 females choose to stay in the labor market if and only if their spouses end up with a low paying jobs, rather than if and only if they themselves end up with high paying jobs as assumed above. Such an alteration to the model will not have any effect on the results, as an almost identical argument to that given above can be used to show that males will still invest more in human capital than either type of female.[9] The intuition is identical to before. Females of both types know that they will never be the only earner within the household, while males know that they may be the only earner in the household with some positive probability. Hence, males have a greater incentive to invest in human capital than either type of female.

A stronger concern regarding the model presented above may be the assumption that males have no preferences or influence over whether or not their spouses stay in the labor market. An alternative to this assumption may be that there also exist two types of males—those that prefer their spouses to work (type-a males) and those that prefer their spouses to stay at home (type-b males). Given such types, it seems reasonable to assume that type-a males generally match with type-1 females and type-b males generally match with type-2 females, to the greatest extent possible.[10] In other words, males who want their spouse to stay home usually match with females who plan to stay home and males who prefer their spouses to work usually match with females who plan to stay in the labor market.

Note that changing the environment in this manner will not qualitatively change the results coming from the model. Type-b males, or males who want their spouses to stay home, will invest the most in human capital, as they clearly have the strongest incentive to do so since they are the most likely to be the sole breadwinner in their households. Type-a males, or males who would prefer their spouses to work, will invest the next most in human capital, as they will still have some probability of being the sole breadwinner in the household as long as there is some probability they will not match with a type-1 female (which could happen if there are more type-a males in the economy than type-1 females or if the matching mechanism is imperfect). Type-1 females, or females that plan on working, will invest less in human capital than either male type, as they are assured of not being the only breadwinner in their household, regardless of what type of male they actually match with. Finally, type-2 females, or the females with the least attachment to the labor force, will invest the least in human capital, as not only will they not be the only breadwinner in their household, but there is a good chance they will not see any earnings benefit at all to their human capital investment.

[9] Proof available from author upon request.

[10] If a greater fraction of females are of type-1 than males are of type-a, the excess type-1 females can be assumed to match with type-b males. Whether such females work after forming such a household can be modelled in a variety of ways, but such details will not generally affect any of the main results of this paper.

3 Discussion and Conclusions

This paper shows that the formation of households by males and females may be an important mechanism for explaining gender differences in labor market human capital investment. In particular, males may have a stronger incentive to invest in labor market human capital than females, since they know that not only will this human capital be put to use, but also that it will be very valuable if they turn out to be the sole earner in their households. By contrast, even those females who plan on working after forming a household, know that it is very unlikely that they will end up being the sole earner in the household. Therefore, even females who plan on working (and indeed continue to work) after forming a household have somewhat less of an incentive to invest in human capital than otherwise identical males.

A final point related to this model is that it rests on a key assumption that there exists an inherent asymmetry between males and females. Namely, there must exist some fraction of females who have a greater likelihood of leaving the labor market after forming a household than males. One justification for this assumption could be that societal norms and traditions cause females to generally be more willing to drop out of the labor market and provide homecare than males. However, this argument does not explain why these social norms exist in the first place. Bjerk and Han (2005) show that there may be labor market reasons for why such norms to arise. In particular, if employers have to pay an adjustment cost when an employee quits, and employers believe females are more likely to quit than equally skilled males, employers will pay females less than their equally skilled male counterparts. If households are then formed assortatively by skill, the male member of the household will then always earn more than his spouse. Therefore, if homecare requirements for a household turn out to be such that it becomes optimal for the household to select one of its members to leave the labor market to attend to these requirements, it will always be optimal for the female member to do so. Such a within household selection process would then confirm employers' beliefs, giving them no incentive to alter their discriminatory behavior. In this way, the social norm of females being the only ones who drop out of the labor force to attend to homecare requirements could be perpetually maintained.

Appendix

Proof that $e_m^* > e_{f1}^* > e_{f2}^*$ from section 2.1.

Given $e_{f2}^* = 0$, it is clear that $e_{f1}^* > e_{f2}^*$ as long as $\omega'(0)$ is assumed to be sufficiently large. Therefore, we only have to prove $e_m^* > e_{f1}^*$. To prove this, say it was not true, or that $e_m^* \leq e_{f1}^*$. If this was the case, then we know $\omega'(e_m^*) \geq \omega'(e_{f1}^*)$ since $\omega'(e) > 0$ and $\omega''(e) < 0$. From equations (4) and (5), we then know that in equilibrium

$$\frac{2}{\pi U'\left(\frac{\omega(e_m^*)+\omega(e_{f1}^*)}{2}\right) + (1-\pi)U'\left(\frac{\omega(e_m^*)}{2}\right)} \geq \frac{2}{U'\left(\frac{\omega(e_m^*)+\omega(e_{f1}^*)}{2}\right)}.$$

Rearranging the above expression and combining terms, the above expression implies

$$(1-\pi)U'\left(\frac{\omega(e_m^*)+\omega(e_{f1}^*)}{2}\right) \geq U'\left(\frac{\omega(e_m^*)}{2}\right).$$

Given the concavity of the U function, for the above expression to hold, it must be true that $\frac{\omega(e_m^*)+\omega(e_{f1}^*)}{2} \leq \frac{\omega(e_m^*)}{2}$, or $\frac{\omega(e_{f1}^*)}{2} \leq 0$ which cannot be true if $e_{f1}^* > e_{f2}^* = 0$ and $w'(0) > 0$ as assumed above. Therefore, it cannot be true that $e_m^* \leq e_{f1}^*$, meaning $e_m^* > e_{f1}^*$.

Proof that $e_m^* > e_{f1}^* > e_{f2}^*$ from section 2.2.

From equations (7) and (8) we know $e_{f1}^* > e_{f2}^*$. Therefore, we once again only have to prove $e_m^* > e_{f1}^*$. As before, say it was not true, or that $e_m^* \leq e_{f1}^*$, meaning $p'(e_m^*) \geq p'(e_{f1}^*)$. From equations (6) and (7), we then know that

$$\begin{aligned}
&\left[\pi\left[p(e_{f1}^*)\left(U\left(\omega^h\right) - U\left(\frac{\omega^h+\omega^\ell}{2}\right)\right) + (1-p(e_{f1}^*))\left(U\left(\frac{\omega^h+\omega^\ell}{2}\right) - U\left(\omega^\ell\right)\right)\right] + \right.\\
&\left.(1-\pi)\left[p(e_{f2}^*)\left(U\left(\omega^h\right) - U\left(\frac{\omega^h+\omega^\ell}{2}\right)\right) + (1-p(e_{f2}^*))\left(U\left(\frac{\omega^h}{2}\right) - U\left(\frac{\omega^\ell}{2}\right)\right)\right]\right]^{-1}\\
&\geq \left[p(e_m^*)\left(U\left(\omega^h\right) - U\left(\frac{\omega^h+\omega^\ell}{2}\right)\right) + (1-p(e_m^*))\left(U\left(\frac{\omega^h+\omega^\ell}{2}\right) - U\left(\omega^\ell\right)\right)\right]^{-1}.
\end{aligned}$$

Rearranging and combining like terms in this equation we get

$$\begin{aligned}
&\left[\left(p(e^*_m) - \pi p(e^*_{f1})\right) - \left(p(e^*_{f2}) - \pi p(e^*_{f2})\right)\right] \left(U\left(\omega^h\right) - U\left(\frac{\omega^h + \omega^\ell}{2}\right)\right) \qquad (9)\\
&+ \Bigg[\left[(1 - p(e^*_m)) - \pi(1 - p(e^*_{f1}))\right] \left(U\left(\frac{\omega^h + \omega^\ell}{2}\right) - U\left(\omega^\ell\right)\right) - \\
&\left[(1 - p(e^*_{f2})) - \pi(1 - p(e^*_{f2}))\right] \left(U\left(\frac{\omega^h}{2}\right) - U\left(\frac{\omega^\ell}{2}\right)\right)\Bigg]\\
&\geq 0.
\end{aligned}$$

If $e^*_m \leq e^*_{f1}$, then the first term in the above expression must be zero or negative. Moreover, the second term must be strictly negative, as $(1 - p(e^*_m)) - \pi(1 - p(e^*_{f1})) \leq (1 - p(e^*_{f2})) - \pi(1 - p(e^*_{f2}))$ and $U\left(\frac{\omega^h+\omega^\ell}{2}\right) - U\left(\omega^\ell\right) < U\left(\frac{\omega^h}{2}\right) - U\left(\frac{\omega^\ell}{2}\right)$ as long as U is strictly concave. Hence, equation (9) cannot hold when $e^*_m \leq e^*_{f1}$. Therefore, it must be true that $e^*_m > e^*_{f1}$.

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