

MATH 60, FALL 2008, PRACTICE TEST II

Please print your name clearly!

Name: _____

Please show all your work, that is explain every step of your solution - it is your work, not just the answer, that is being evaluated. When asked to prove a statement, make sure to provide reasoning behind each claim you are making in the process of the proof. The use of calculators or any other electronic devices is prohibited during the test. You are also not allowed to use any study materials except for those provided to you during the test. Good luck!

Problem 1. (35 points) Let $V = \text{span}\{\sin x, \cos x\}$ be a two-dimensional subspace of $C[-\pi, \pi]$, the vector space of all continuous functions on the interval $[-\pi, \pi]$. For each $f, g \in V$, define an inner product

$$\langle f, g \rangle = \int_{-\pi}^{\pi} f(x)g(x)dx.$$

Let us view V as an inner product space with respect to \langle , \rangle .

a) (20 points) Prove that $\sin x$ and $\cos x$ are orthogonal in V , and compute their norms with respect to \langle , \rangle . Show all work.

b) (15 points) Use Pythagorean Law to compute the norm of $f(x) = 2 \cos x + 3 \sin x$. Show all work.

Problem 2. (35 points) Let $V = \text{span}\{e^x, x^2\}$ be a two-dimensional subspace of $C^1(-\infty, \infty)$, the space of all differentiable functions on the entire real line. For each $f(x) \in V$ define

$$L(f) = f'(x) - f(x).$$

a) (15 points) Prove that $L : V \rightarrow C^1(-\infty, \infty)$ is a linear transformation.

b) (20 points) Find the image and the kernel of L , and indicate their respective dimensions. Show all work.

Problem 3. (30 points) Recall that \mathbb{P}_2 is the vector space of all polynomials of degree less or equal to two. Let

$$p_1(x) = x^2 + 2x - 1, \quad p_2(x) = x^2 - 2x + 1, \quad p_3(x) = x,$$

and let

$$q_1(x) = x - 1, \quad q_2(x) = x + 1, \quad q_3(x) = x^2.$$

It is not difficult to notice that p_1, p_2, p_3 and q_1, q_2, q_3 are two different bases for \mathbb{P}_2 (you do not need to prove this). Please compute the transition matrices from p_1, p_2, p_3 to q_1, q_2, q_3 and from q_1, q_2, q_3 to p_1, p_2, p_3 , indicating which one is which. Show all work.