

Examples of bounds on Frobenius number for WR lattices

Erdős, Graham (1972):

$$\mathcal{F} \leq 2a_N \left\lceil \frac{a_1}{N} \right\rceil - a_1. \quad (1)$$

Vitek (1975):

$$\mathcal{F} \leq \left\lceil \frac{(a_2 - 1)(a_N - 2)}{2} \right\rceil - 1. \quad (2)$$

Selmer (1977):

$$\mathcal{F} \leq 2a_{N-1} \left\lceil \frac{a_N}{N} \right\rceil - a_N. \quad (3)$$

Beck, Diaz, Robins (2002):

$$\mathcal{F} \leq \frac{1}{2} \left(\sqrt{a_1 a_2 a_3 (a_1 + a_2 + a_3)} - a_1 - a_2 - a_3 \right). \quad (4)$$

Fukshansky, Robins (2005, WR case):

$$\mathcal{F} \leq \left\lceil \frac{(N-1)R_{\mathbf{a}}}{\|\mathbf{a}\|} \sum_{i=1}^N a_i \sqrt{\|\mathbf{a}\|^2 - a_i^2} + 1 \right\rceil \leq \left\lceil \frac{(N-1)^2 \sum_{i=1}^N a_i \sqrt{\|\mathbf{a}\|^2 - a_i^2} + 1}{(\|\mathbf{a}\|^{N-2} \omega_{N-1})^{\frac{1}{N-1}}} + 1 \right\rceil. \quad (5)$$

$N = 4$:

| <i>4-tuple</i> \mathbf{a} | $\lambda_{\mathbf{a}}$ | <i>min</i> (1) - (4) | (5) | (5) <i>w/</i> $R_{\mathbf{a}}$ |
|-----------------------------|------------------------|----------------------|----------|--------------------------------|
| 9337, 9961, 11593, 67367 | $\sqrt{1802}$ | 91235853 (4) | 10995433 | 4510642 |
| 33199, 38351, 47759, 152057 | $\sqrt{3218}$ | 1346684400 (4) | 55055950 | 23305193 |

$N = 5$:

| <i>5-tuple</i> \mathbf{a} | $\lambda_{\mathbf{a}}$ | <i>min</i> (1) - (4) | (5) | (5) <i>w/</i> $R_{\mathbf{a}}$ |
|---|------------------------|----------------------|-------------|--------------------------------|
| 39221, 46967, 47869, 62839, 206749 | $\sqrt{524}$ | 1719019240 (4) | 66231577 | 21439724 |
| 1867558, 2348176, 2918749, 5249843, 26695349 | $\sqrt{5591}$ | 4778060891200 (4) | 14595157176 | 4650337779 |

$N = 6$:

| <i>6-tuple</i> \mathbf{a} | $\lambda_{\mathbf{a}}$ | <i>min</i> (1) - (4) | (5) | (5) <i>w/</i> $R_{\mathbf{a}}$ |
|--|------------------------|----------------------|--------------|--------------------------------|
| 6595, 90709, 110483, 121833, 147472, 462217 | $\sqrt{209}$ | 1015946371 (1) | 168600688 | 44444915 |
| 5958323, 14864655, 19945128, 28191201, 28507523, 117697394 | $\sqrt{1915}$ | 134180083643479 (4) | 104669816535 | 26825942777 |